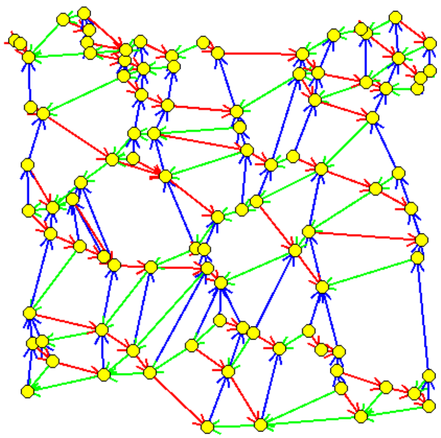
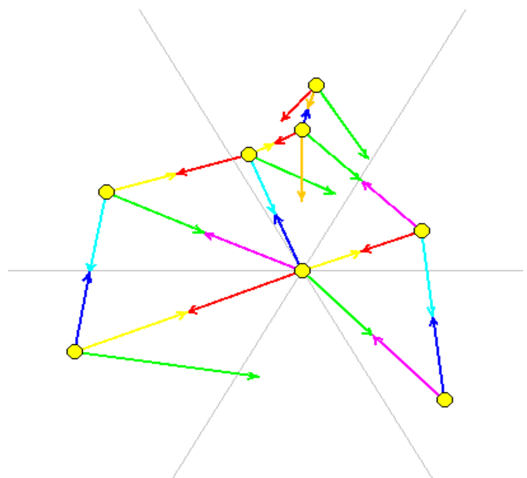
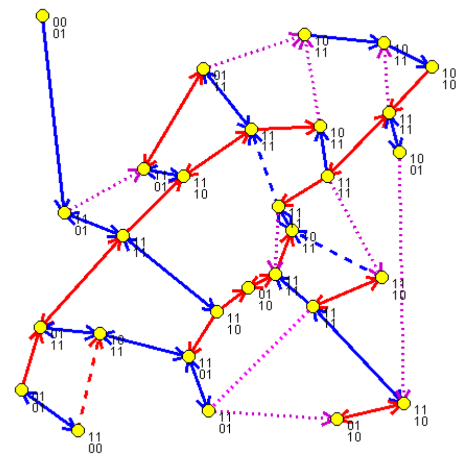
10-15 déc. 2017 Aussois (France)

Etirement de quelques graphes géométriques







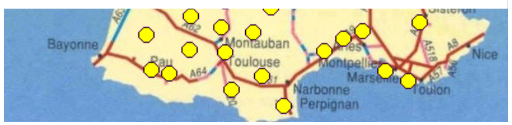
Nicolas Bonichon

LaBRI

université de BORDEAUX

Qu’est-ce qu’un bon réseau ?

- *Sensor Networks*
- Manet (mobile agent)
- Vanet (vehicular agent)
- Ad Hoc Networks
- Mesh Networks
- ...



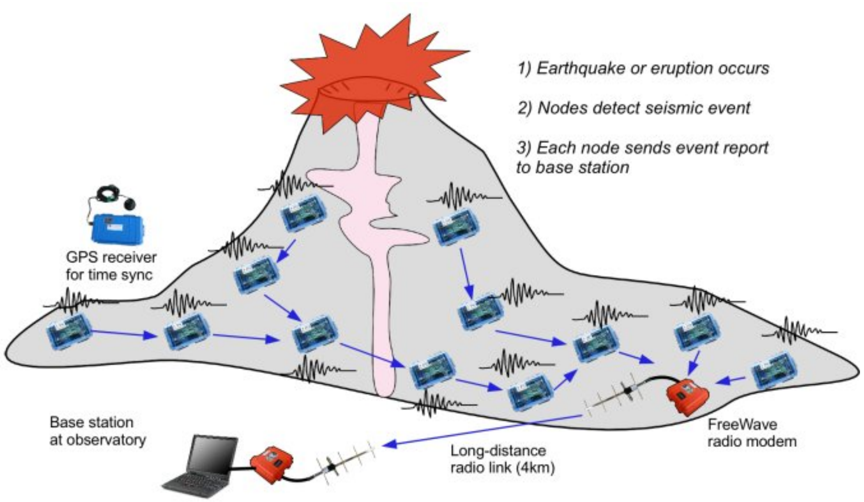
$$\text{Etirement} := \max_{a,b} \frac{\text{distance dans le réseau}_{a\to b}}{\text{distance à vol d'oiseau}_{a\to b}}$$

Nombre d’arêtes ou somme des longueurs des arêtes

Degré maximal : taille du plus grand rond-point

Planaire : ni pont ni tunnel

Facilement Routable



www.2m40.com

50 accidents en 5 ans

Modélisation du problème (but du jeu)

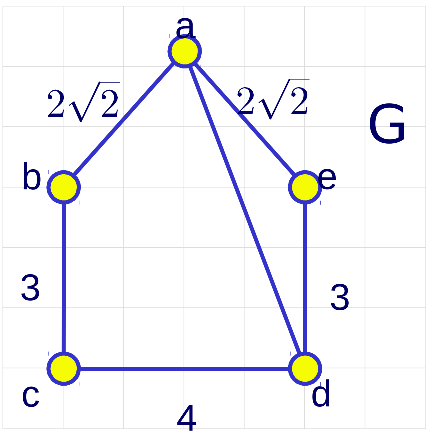
Etant donné un ensemble **V** de points dans le plan :

Construire un réseau (*Graphe*) **G** constitué de segments (*arêtes*) reliant ces points, tel que

1) l'étirement soit faible
2) [les segments ne se croisent pas]

Détour (s,t) : $\frac{d_G(s,t)}{||st||}$

Etirement (G): $\max_{s \neq t}$ Détour(s,t)



Exemple :

$d_G(b,d) = 3 + 4$, $||bd|| = 5$

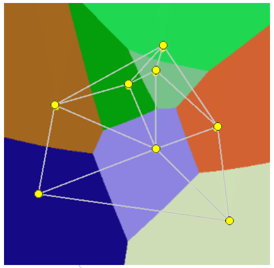
$d_G(b,e) = 4\sqrt{2}$, $||be|| = 4$.

G a un étirement de $\sqrt{2}$.

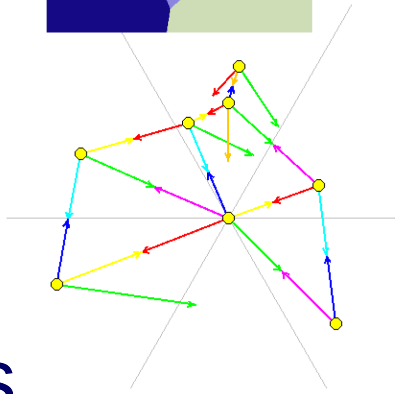
Un graphe est un t-spanner si son étirement est au plus *t*.

Spanners

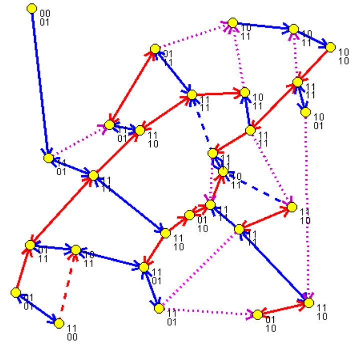
- Triangulations de Delaunay



- Theta-Graphs



- Bounded degree planar spanners

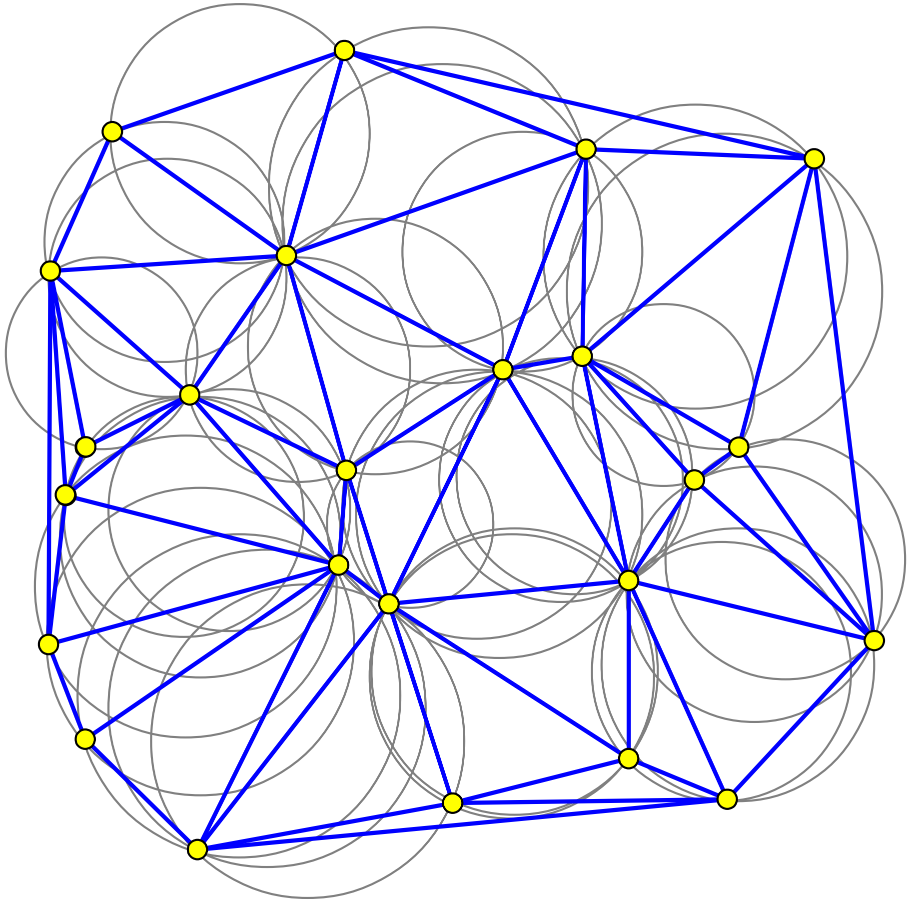


[*Robert* Delaunay]

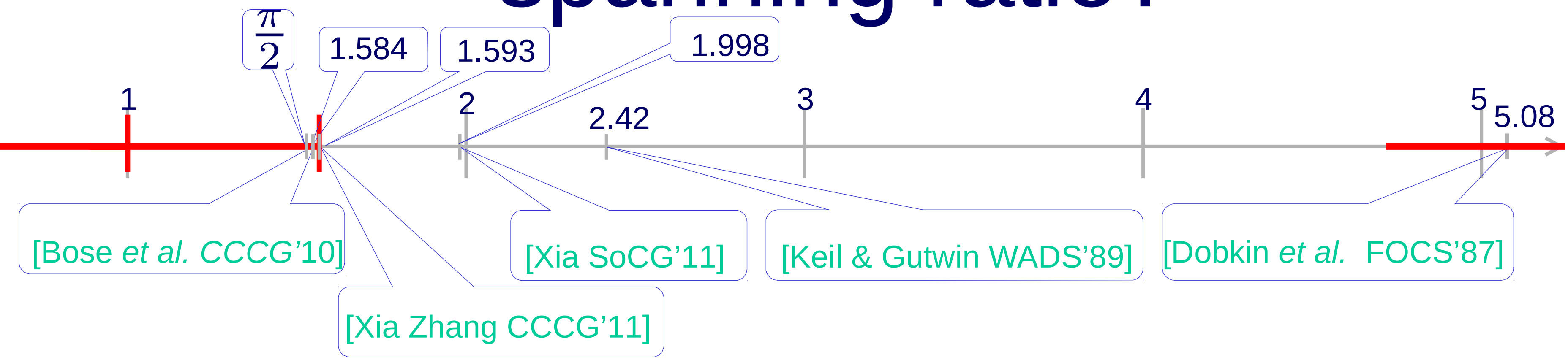
Delaunay Triangulation

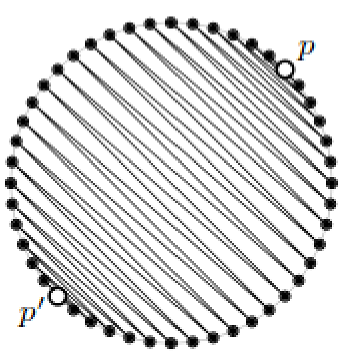
[*Boris* Delaunay'34]

draw triangle x, y, z if its circumscribed circle doesn't contain other points.

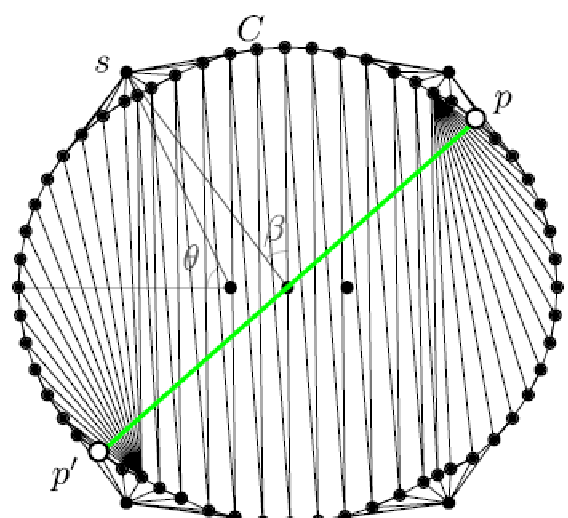


Delaunay triangulation spanning ratio?

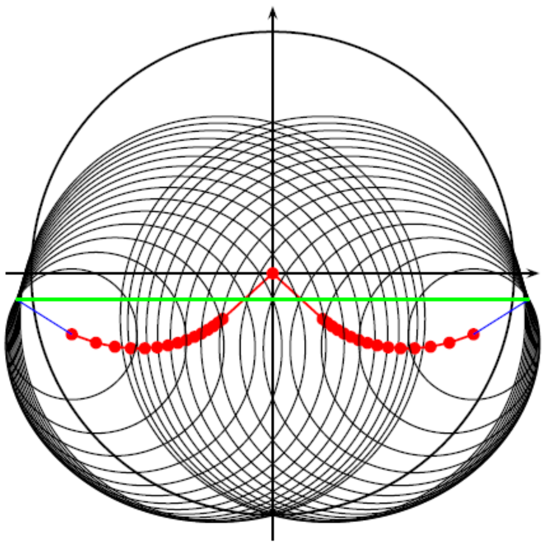




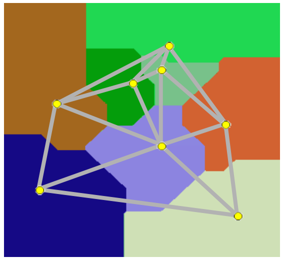
$\pi/2 \approx 1.57079$



1.584



1.593

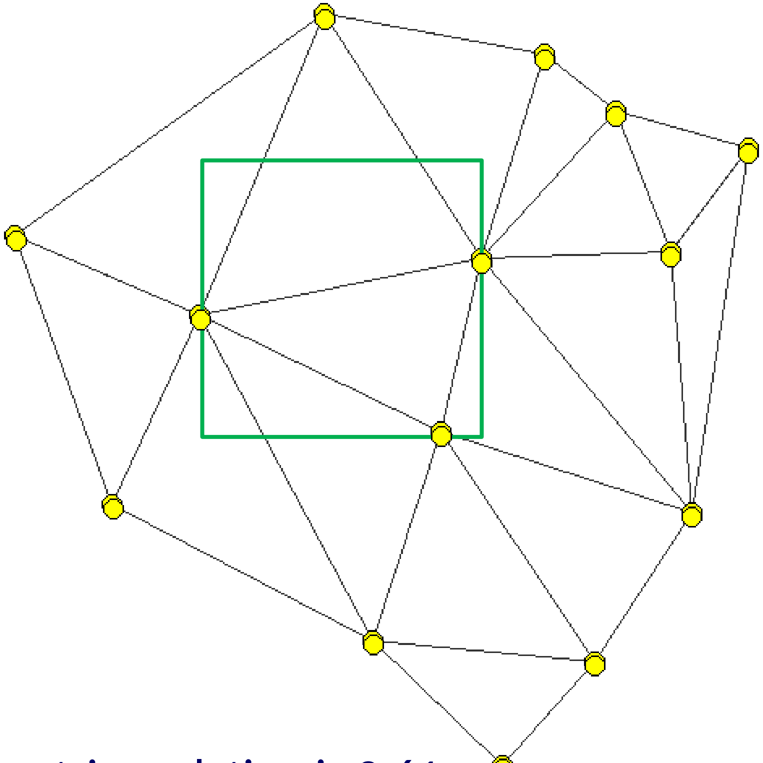


L_∞-Delaunay Triangulation

[Chew SoCG'86]

$$d_\infty((x_1,y_1),(x_2,y_2)) = \max(|x_1-x_2|,|y_1-y_2|)$$

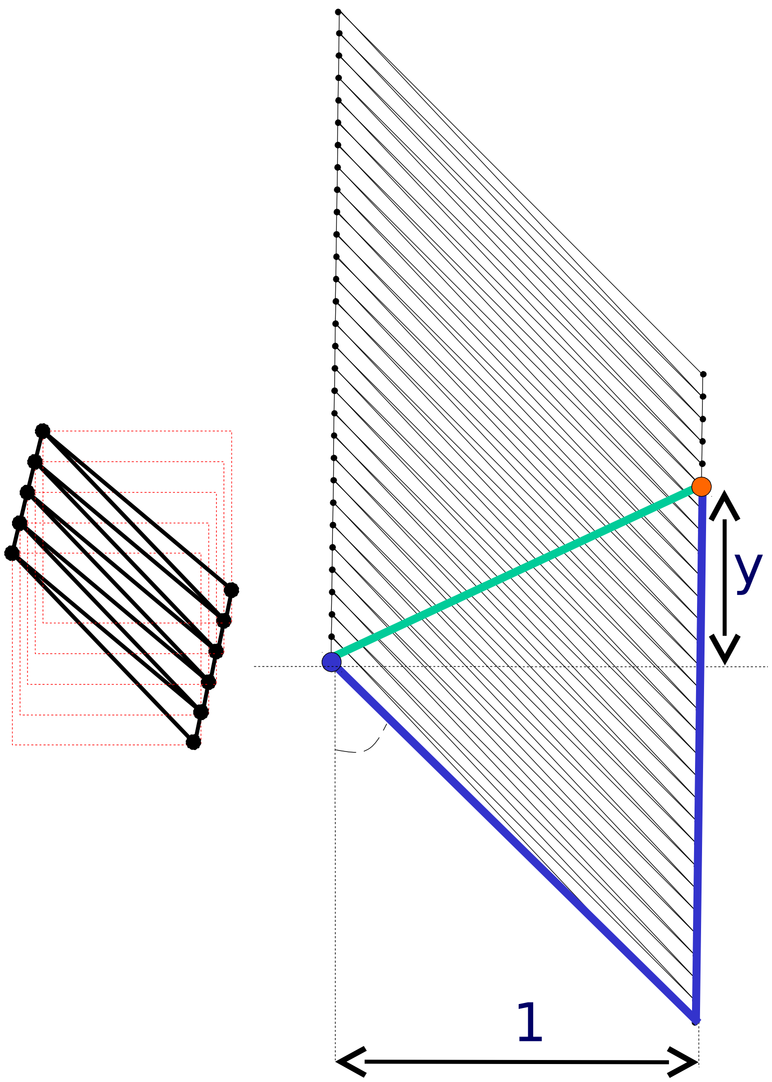
draw triangle x, y, z if its circumscribed circle doesn't contain other points.

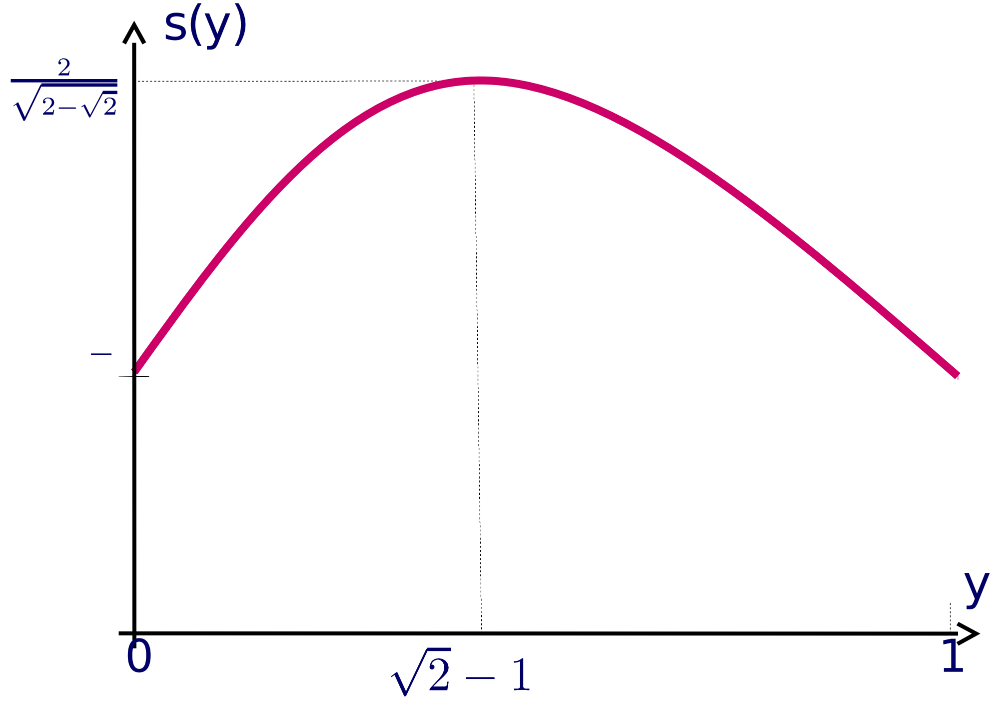


Thm [B. Gavoille Hanusse Perković 10]: L_∞-Delaunay triangulation is 2.61-spanner.

Stretch ≥ 2.61

$$s(y) = \frac{\sqrt{2}+1+y}{\sqrt{1+y^2}}$$

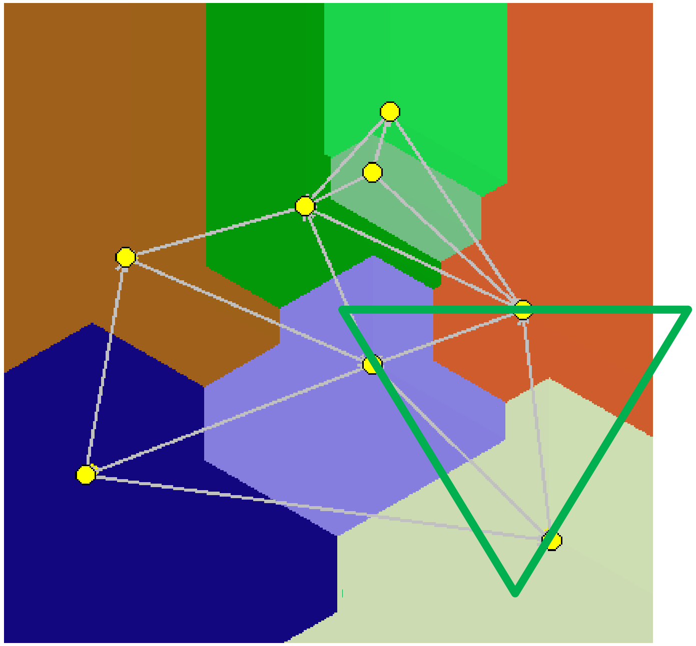




TD-Delaunay triangulations

[Chew 89]

TD : Triangular Distance



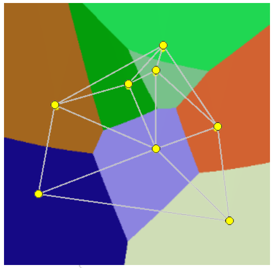
Thm [Chew 89] : TD-Delaunay triangulations are planar 2-spanners.

strech
*p*gone-Delaunay

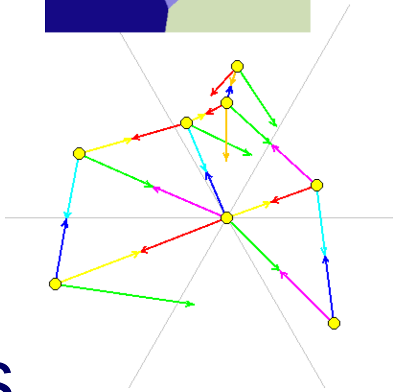
*p*gone	Lower bound	Upper bound
3	2 [Chew 89]	2 [Chew 89]
4	2.61	3.16 [Chew 86] 2.61 [B. Gavoille Hanusse Perković 12]
5	1.96	?
6	2	2 [Dennis, Perković,Türkoğlu 17+]
p>6	?	?
∞	1.593 [Xia Zhang 11]	1.998[Xia 11]

Spanners

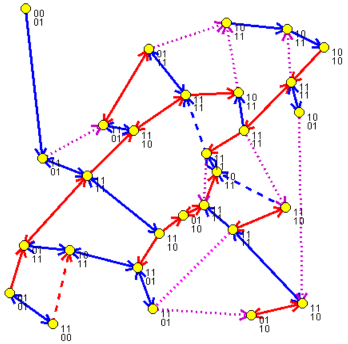
- Triangulations de Delaunay

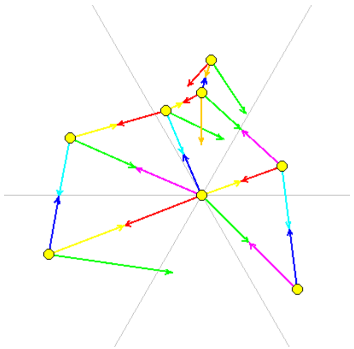


- Theta-Graphs



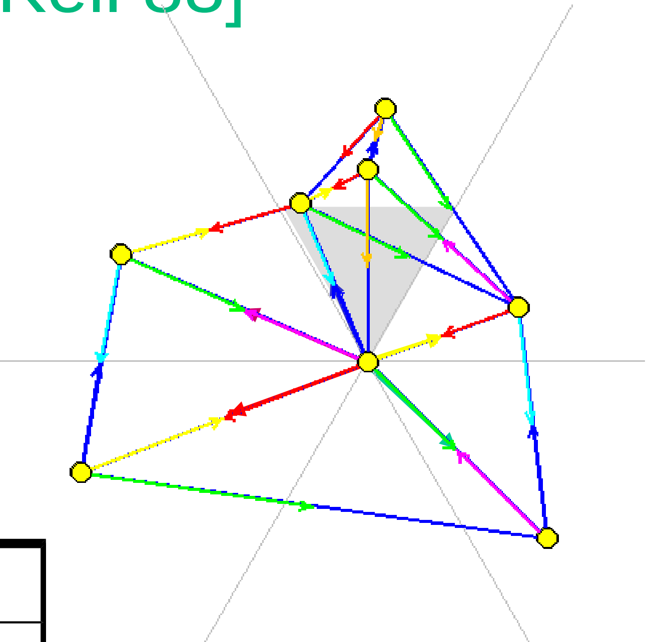
- Bounded degree planar spanners





θ-graphe

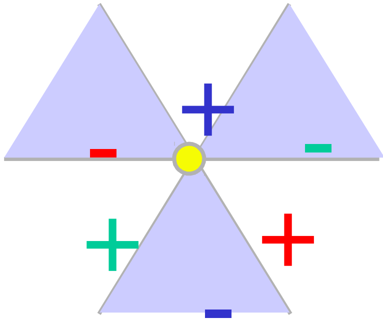
[Clarkson 87][Keil 88]



- Soit k ≥ 2, θ = 2 π/k.
- Le plan autour de chaque sommet *s* est divisé en *k* cônes.
- Chaque somme s choisit un voisin par cône.

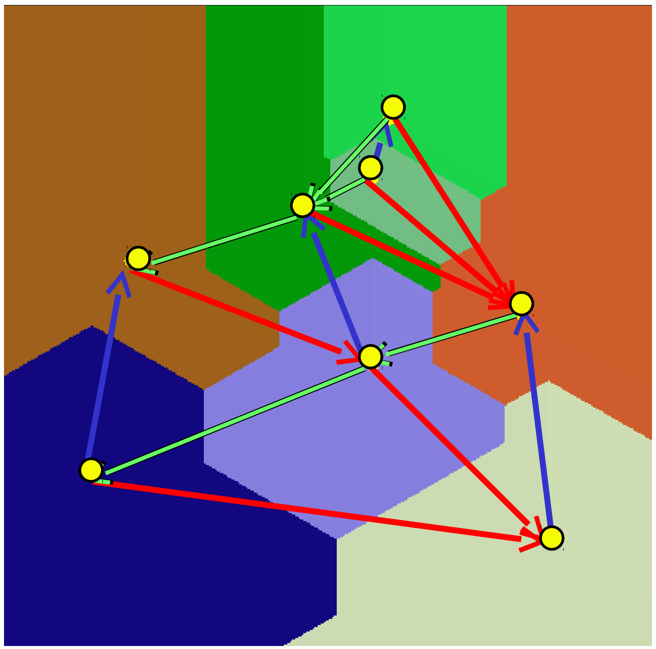
reference	k	stretch
[El Molla 09]	2,3	∞
[Barba et al. 13+]	4	237
[Bose et al. 13+]	5	9.96
[B. Gavoille Ilcinkas Hanusse 10]	6	2
[Clarkson 87][Keil 88]	≥ 7	$\frac{1}{1-2\sin(\pi/k)}$

k	stretch
7	7.56
8	4.26
…	…
12	2.07
13	1.91

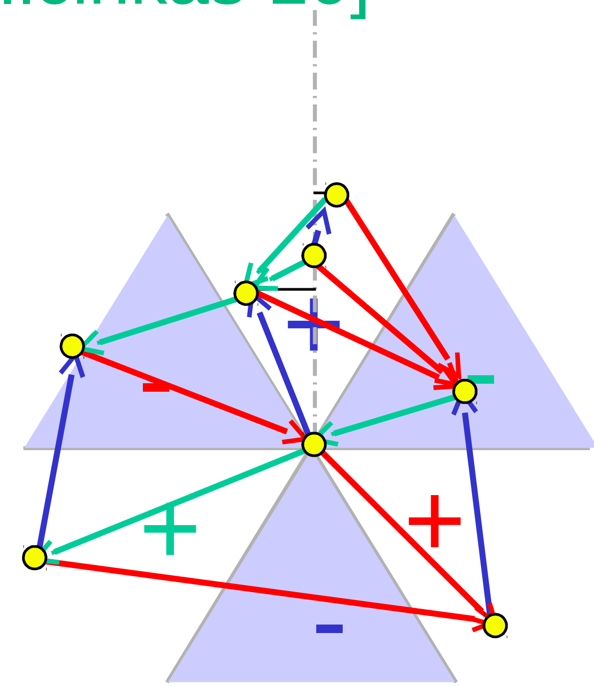


½θ-graphe

[B. Gavoille Hanusse Ilcinkas 10]



• :



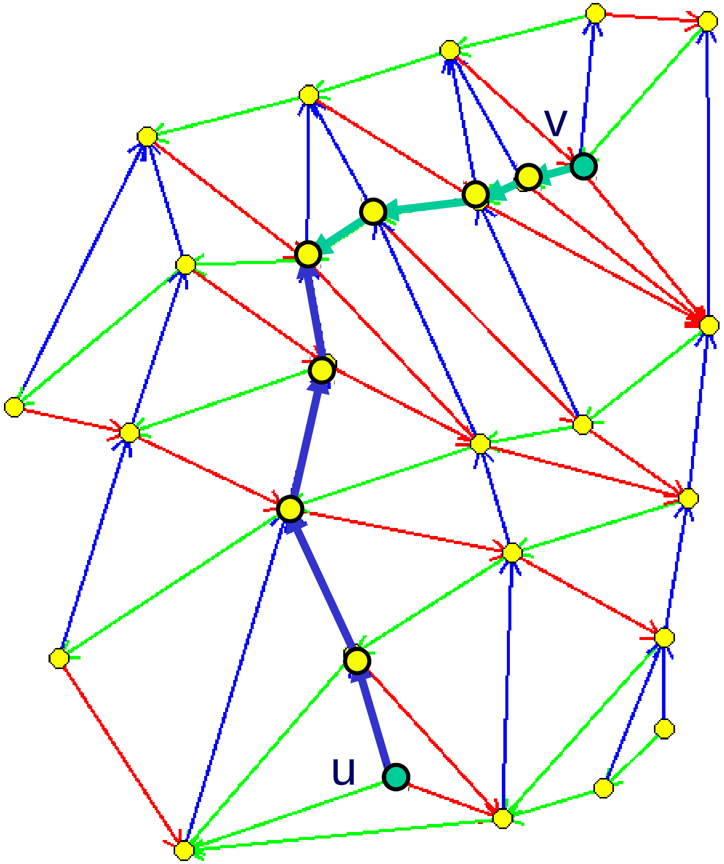
Thm [B. Gavoille Hanusse Ilcinkas 10]:

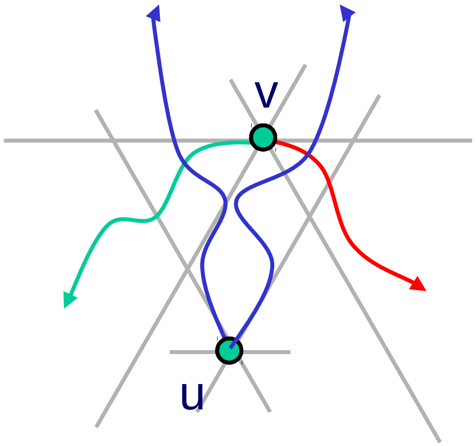
½θ-Graphe ⇔ TD-Delaunay (+structure)

Corollaire :

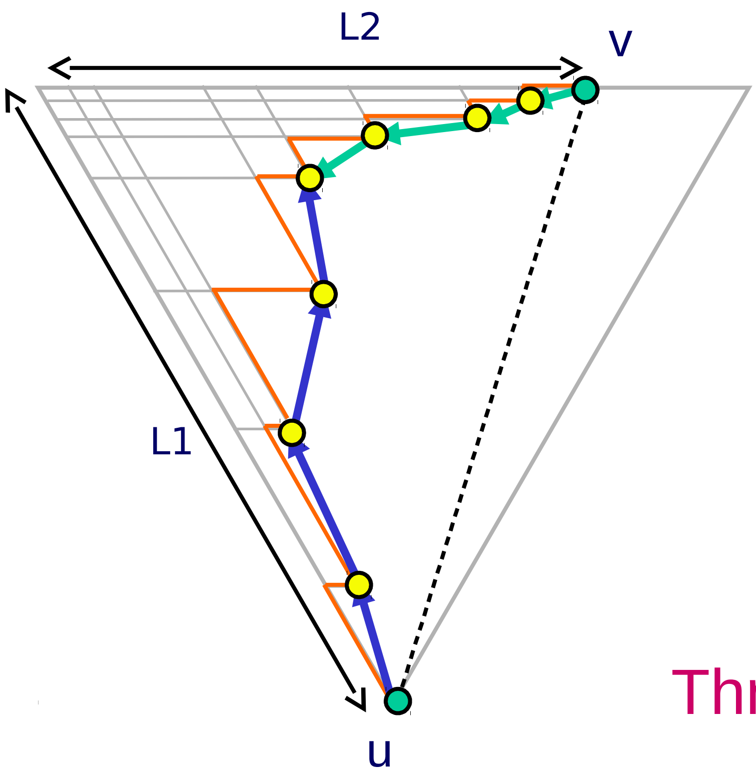
- Les ½θ-Graphes sont des 2-spanners planaires
- θ_6 est un 2-spanner.

stretch?





Stretch?



$$|chemin(u,v)| \leq \ell_1 + \ell_2$$

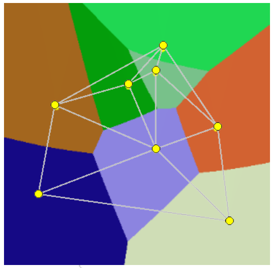
$$||uv|| = \sqrt{\ell_1^2 + \ell_2^2 - \ell_1 \ell_2}$$

$$||uv|| \geq \frac{\ell_1 + \ell_2}{2}$$

Thm: Stretch ≤ 2

Spanners

- Triangulations de Delaunay



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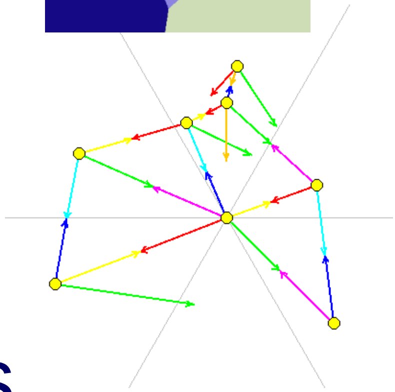
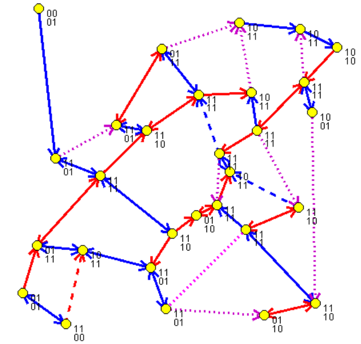


- Bounded degree planar spanners

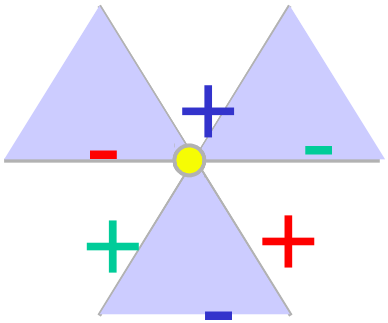


Spanners planaires degré borné

Paper	DegMax	Planar	stretch factor
Folklore	2	X	∞
[Salowe 94]	4	X	C > 405
[Das and Heffernan 96]	3	X	
[Bose *et al.* 05]	27	√	10.016

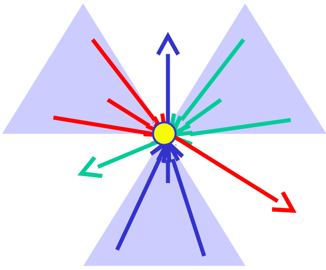
Spanners planaires degré borné

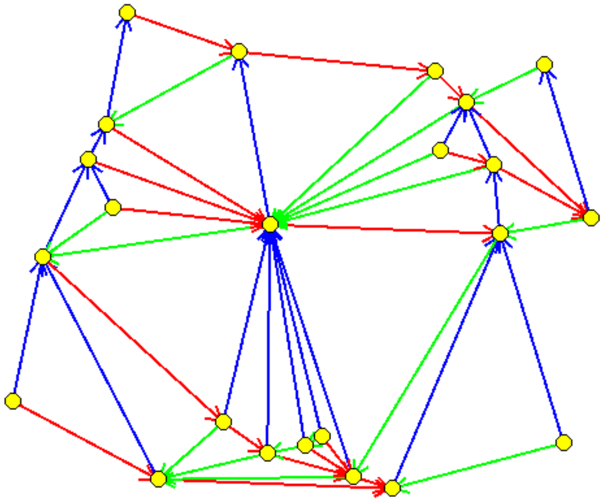
Paper	DegMax	Planar	stretch factor
Folklore	2	X	∞
[Salowe 94]	4	X	C > 405
[Das and Heffernan 96]	3	X	
[Bose *et al.* 05]	27	√ ○	10.016
[Li and Wang 04]	23	√ ○	7.79
[Bose *et al.* 09]	17	√ ○	28.54
[Kanj and Perković 08]	14	√ ○	3.53
[B. Gavoille Hanusse Perković 10]	12	√ △	6
[Bose *et al.* 12]	6	√ ○	81.66
[B. Kanj Perković Xia 14]	4	√ □	16.9 < C <157
[Kanj, Perković, Türkoğlu 16]	4	√ □	20

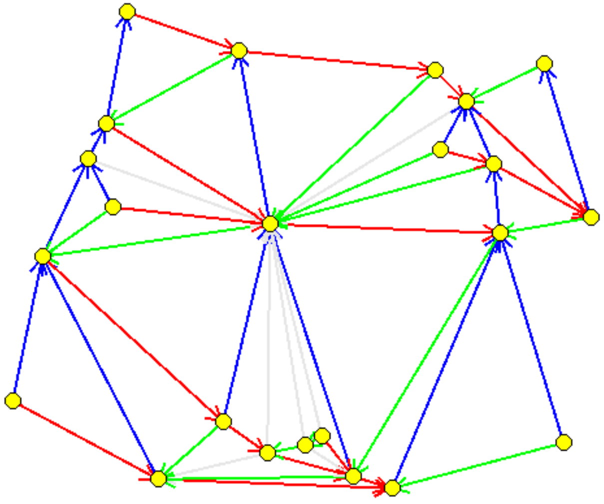


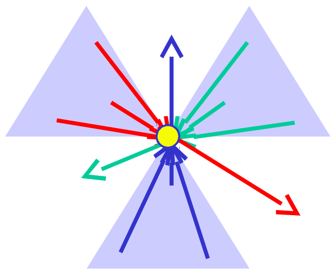
H_{12}: étirement 6 et $deg_{max} \leq 12$

- **H_{12}**: pour chaque cône (-) on garde au plus 3 arêtes :
  - La plus “courte”
  - La plus à gauche
  - La plus à droite
- Degré :









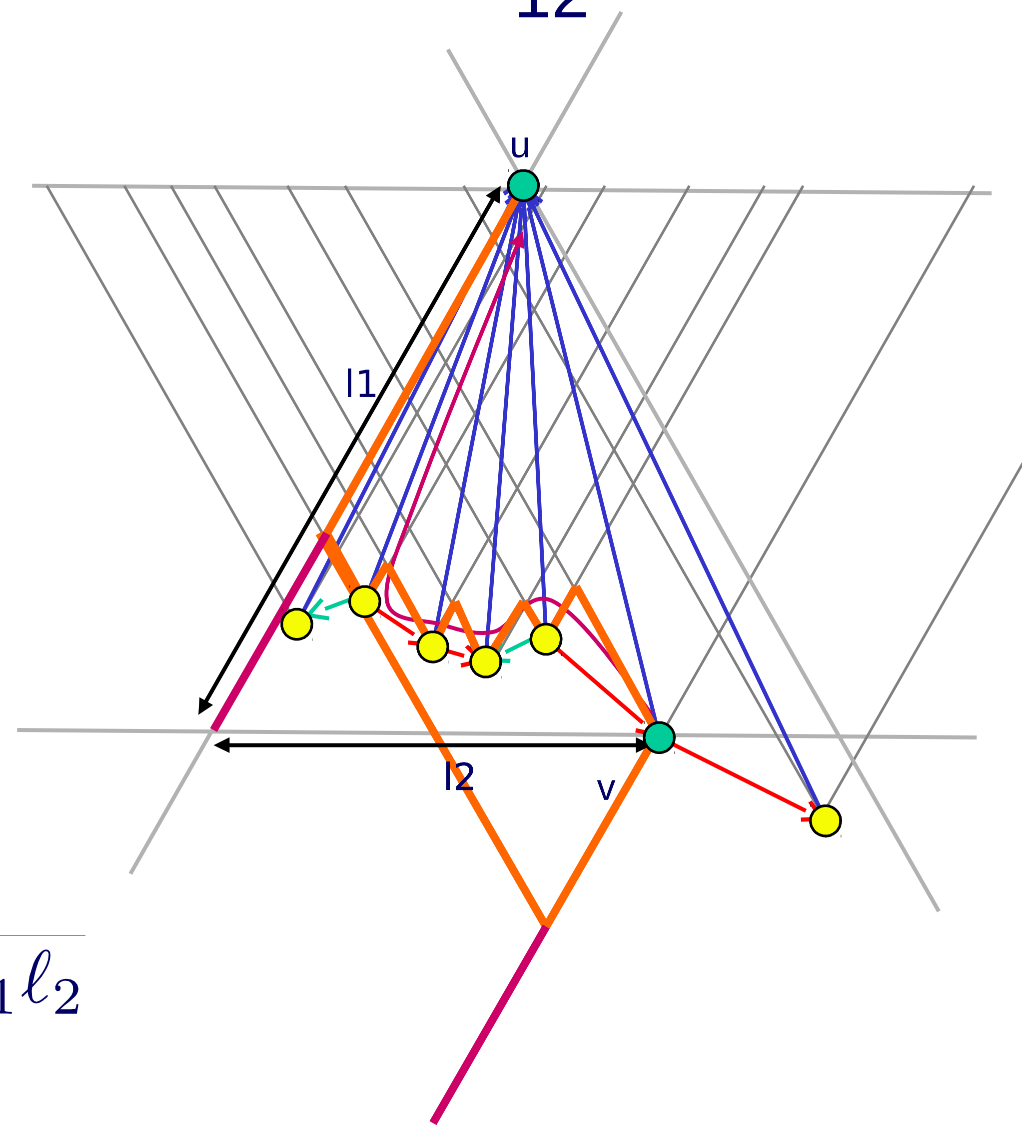
Étirement de H_{12}

- Idée: chaque arête de H_ifn est replacée par un chemin 3 fois plus long dans H_{12} (S) $\Rightarrow$ H_{12} a un étirement <= 6.
- Pour chaque arête uv supprimée, on considère le chemin canonique :
  - Faire le tour de la face en utilisant l'arête courte.

$$|Path(u,v)| = l_1 + 2l_2$$

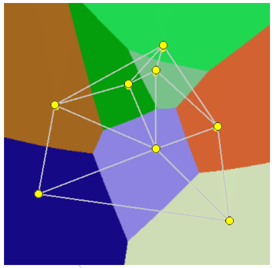
$$d(u,v) = \sqrt{\ell_1^2 + \ell_2^2 - \ell_1\ell_2}$$

$$\frac{|Path(u,v)|}{d(u,v)} \leq 3$$

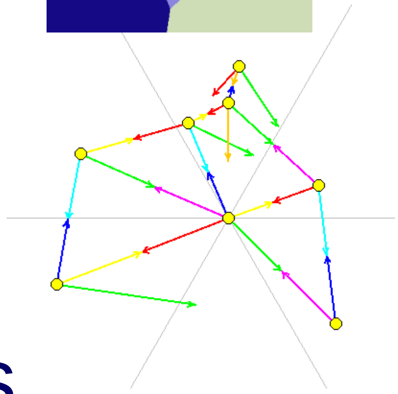


Conclusion

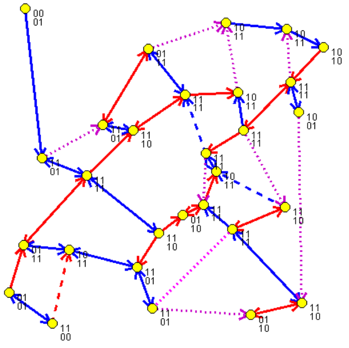
- Triangulations de Delaunay



- Theta-Graphs



- Bounded degree planar spanners



- Tomorrow: routing in spanners