Geometric methods for the conception of optical components in non-imaging optics

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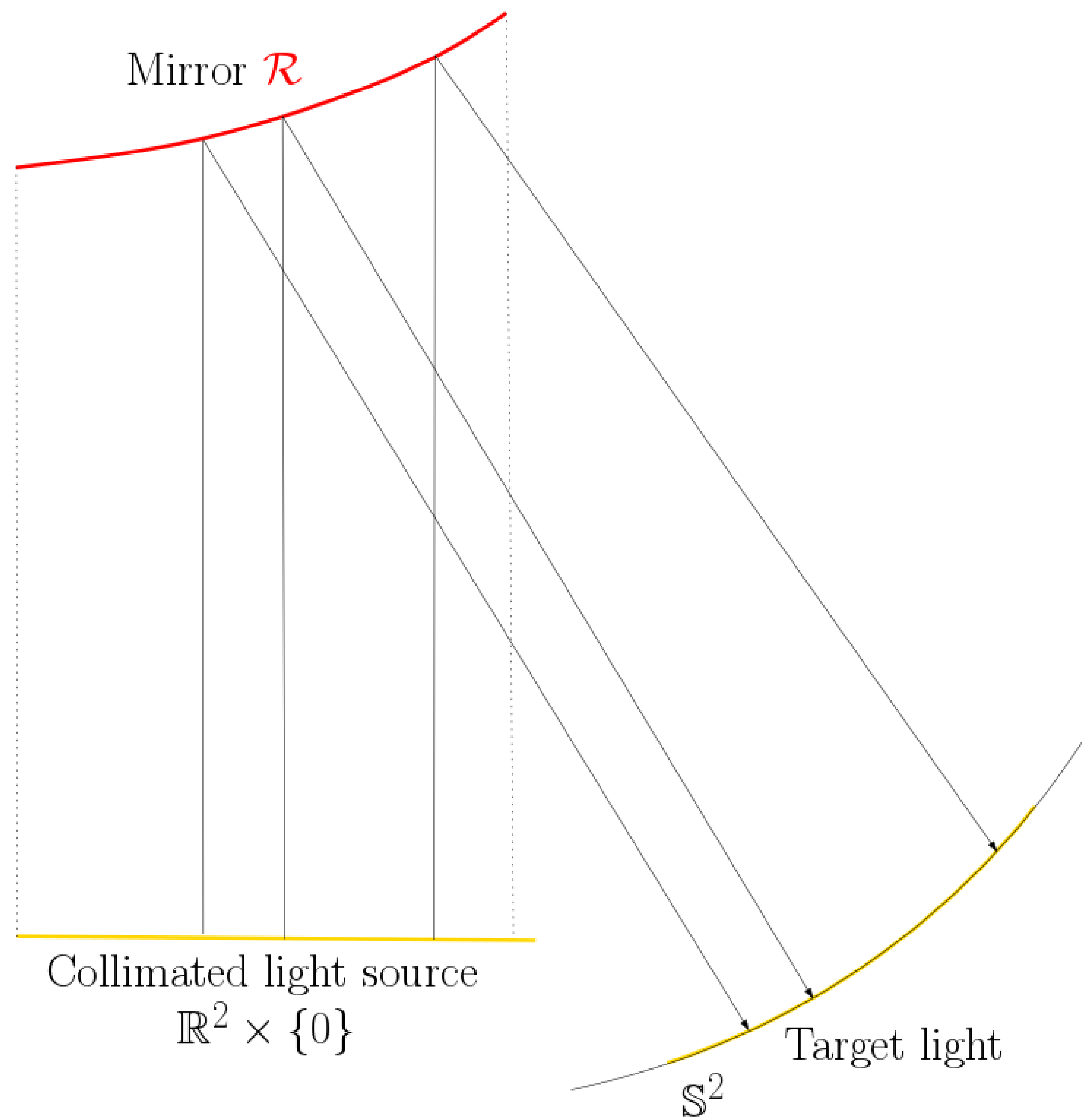
Non-imaging optics

Goal: build optical components (mirrors or lenses) given

- a light *source* distribution
- a **prescribed** *target* distribution

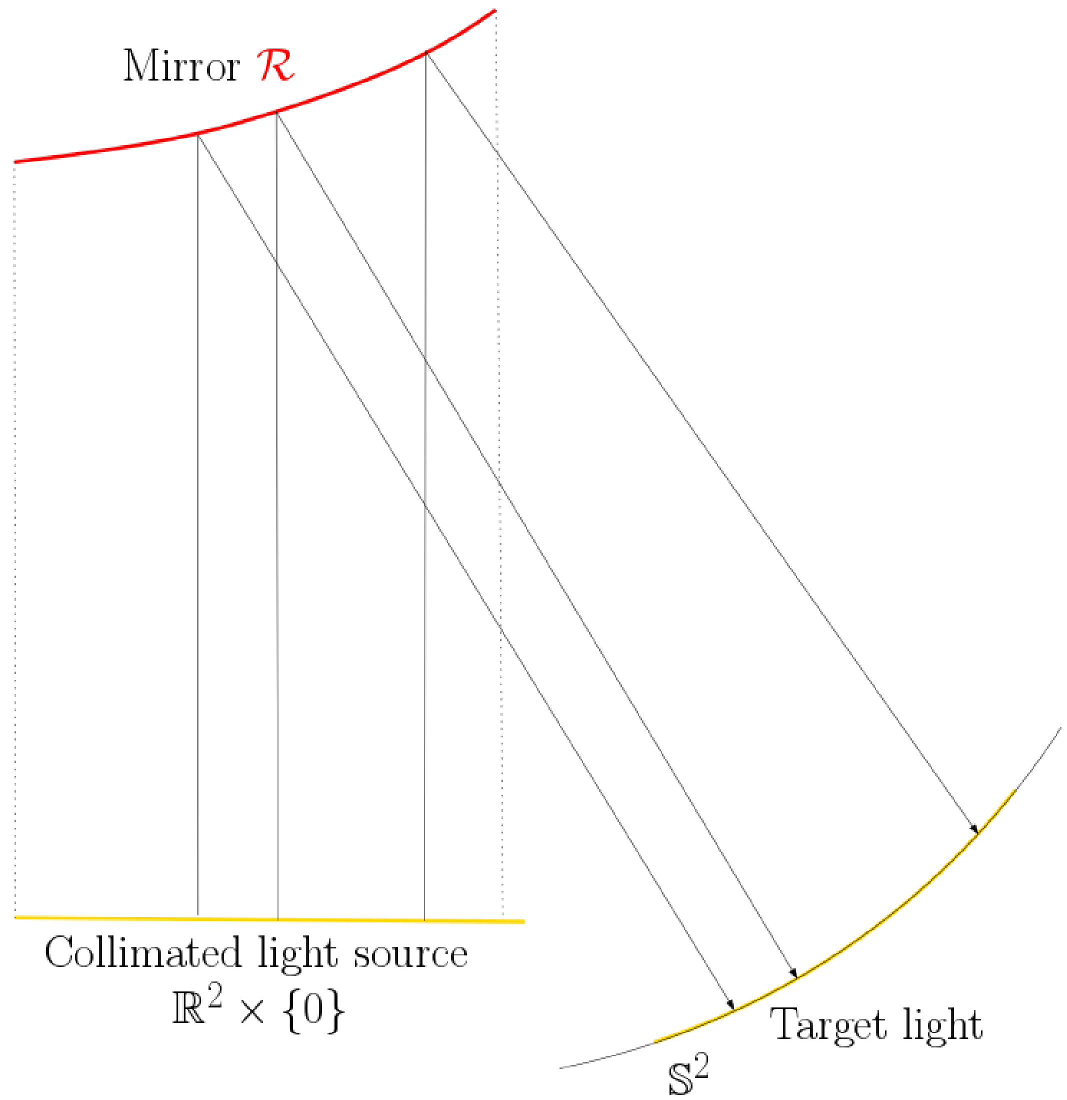
Applications:

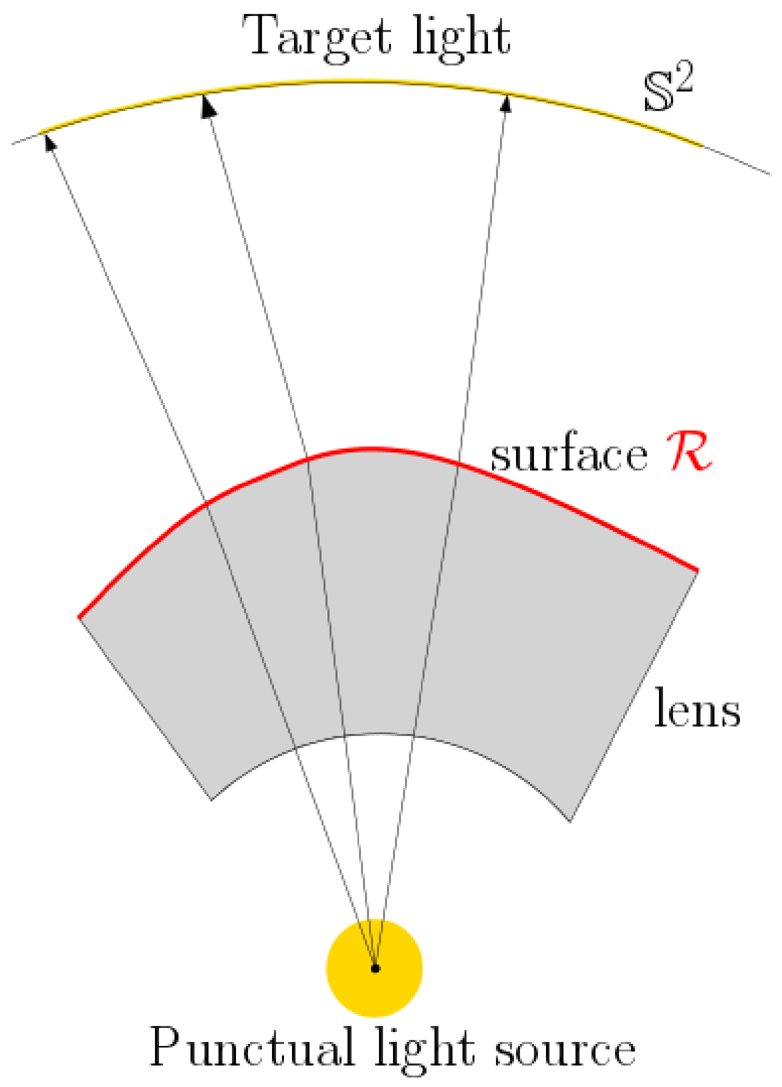
- Hydroponic agriculture
- Public lighting
- Car beams
- Caustic design



Non-imaging problems

- Source can be **collimated** or **punctual**
- Target can be at
  - infinity: directions in $\mathbb{S}^2 \to$ **Far-Field**
  - finite distance: points in $\mathbb{R}^3 \to$ **Near-Field**

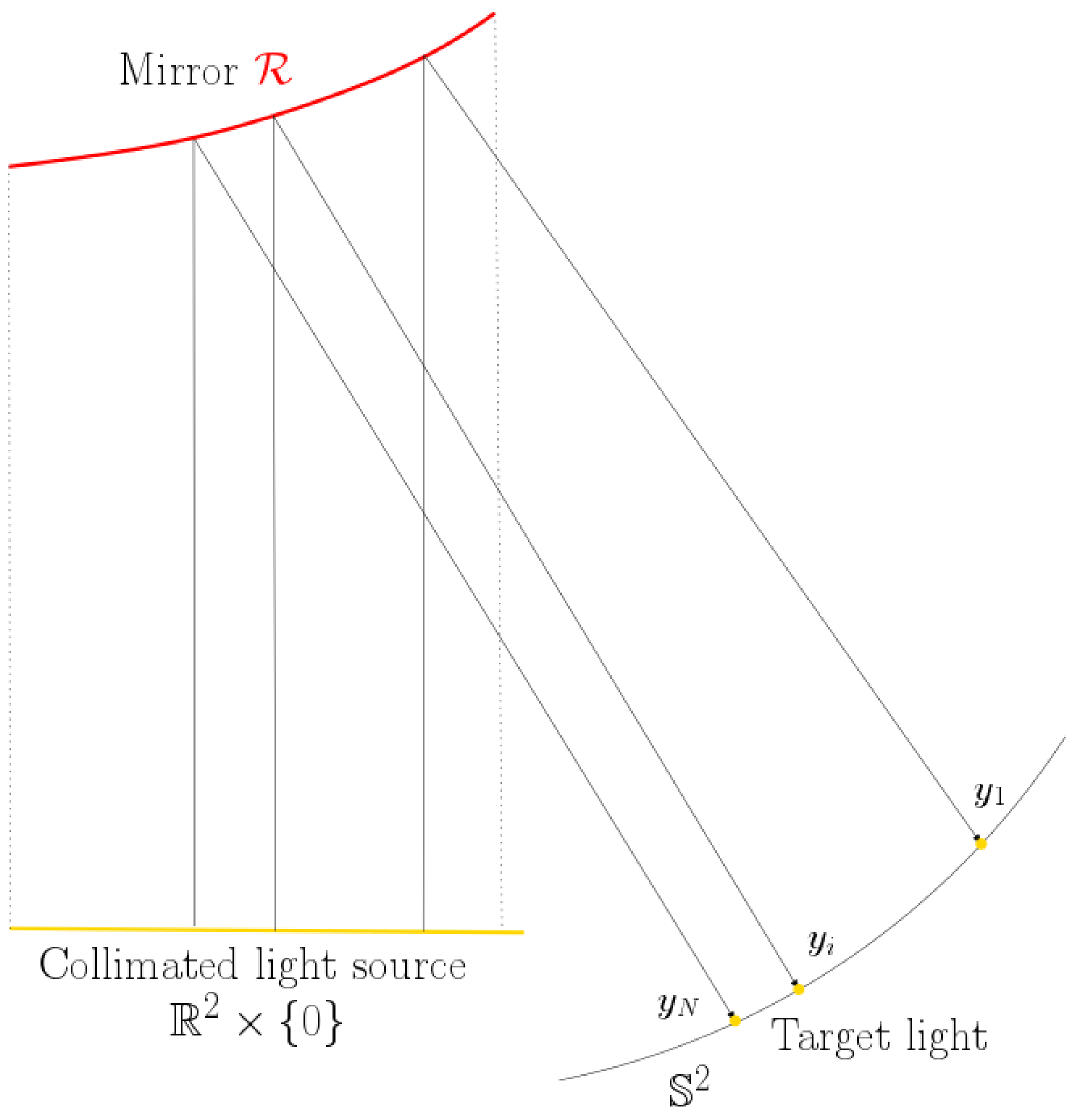


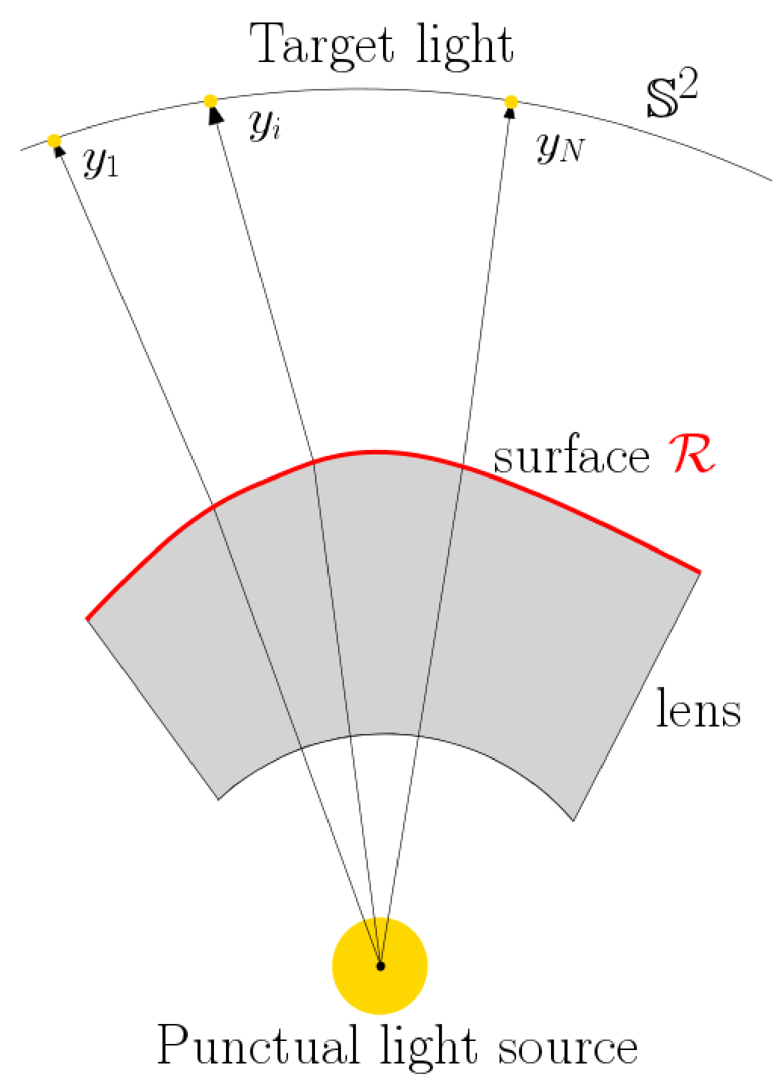


Non-imaging problems

Semi-discrete setting:

- Source is a probability density ρ on $X \subset \mathbb{R}^3$
- Target is a *discrete* measure $\sigma = \sum_{i=1}^{N} \sigma_i \delta_{y_i}$ supported on $Y \subset \mathbb{R}^3$





Outline

Part 1: General framework

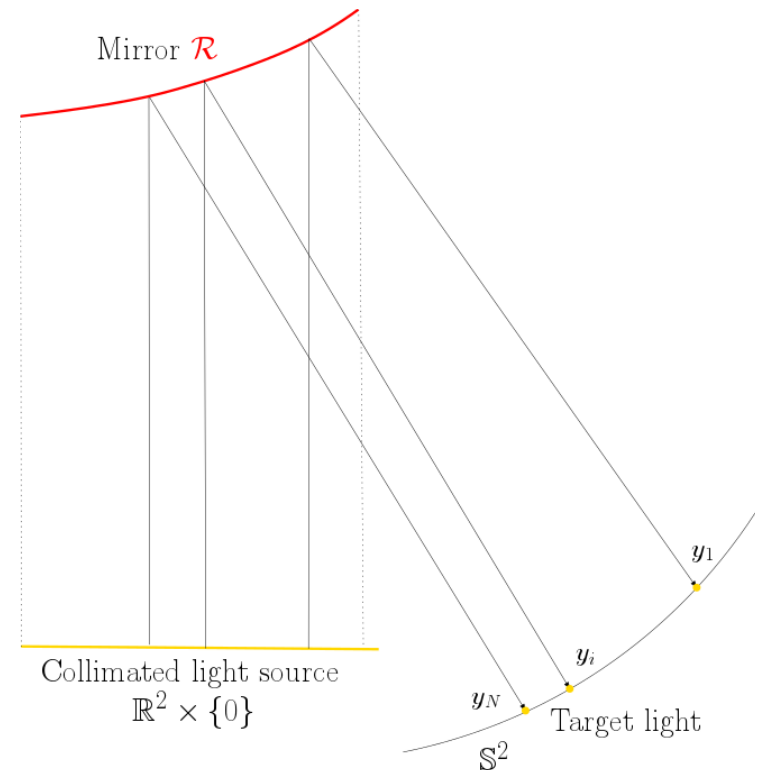
- Setting 1: Collimated Source Mirror
- Setting 2: Collimated Source Lens
- Setting 3: Point Source Mirror
- Setting 4: Point Source Lens

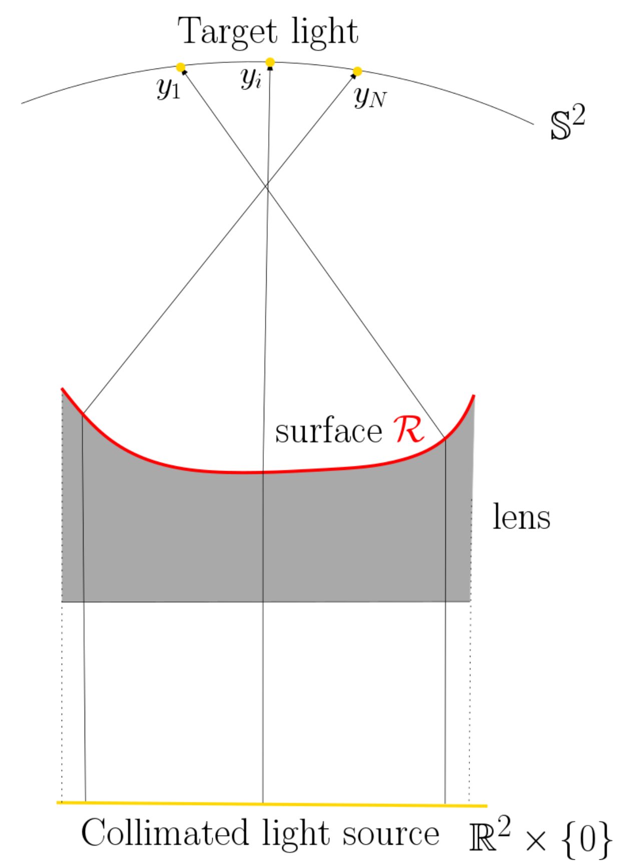
Part 2: Damped Newton algorithm

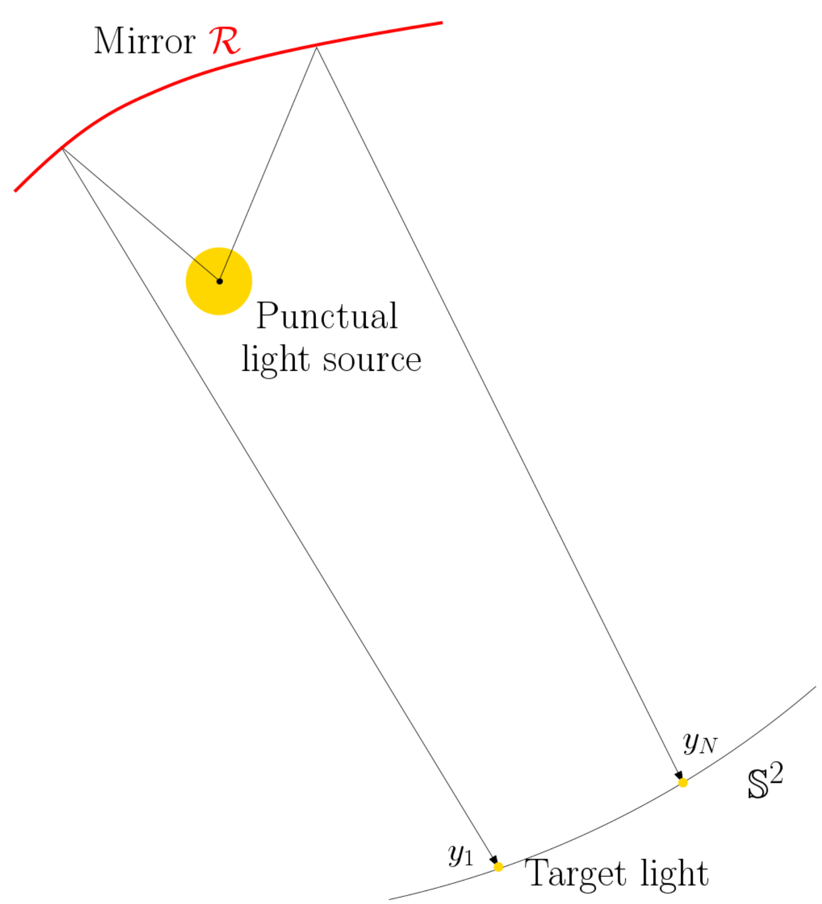
- Description
- Convergence analysis
- Numerical results

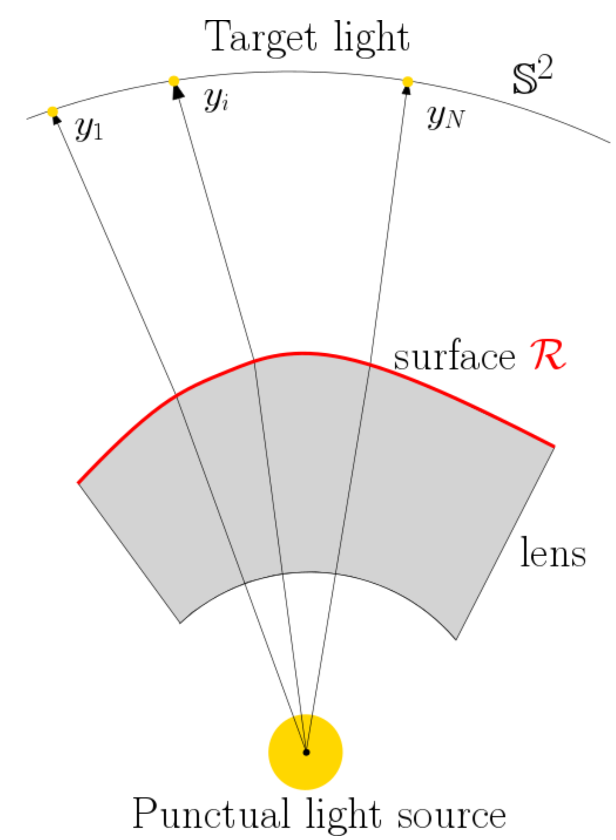
General framework

The different settings



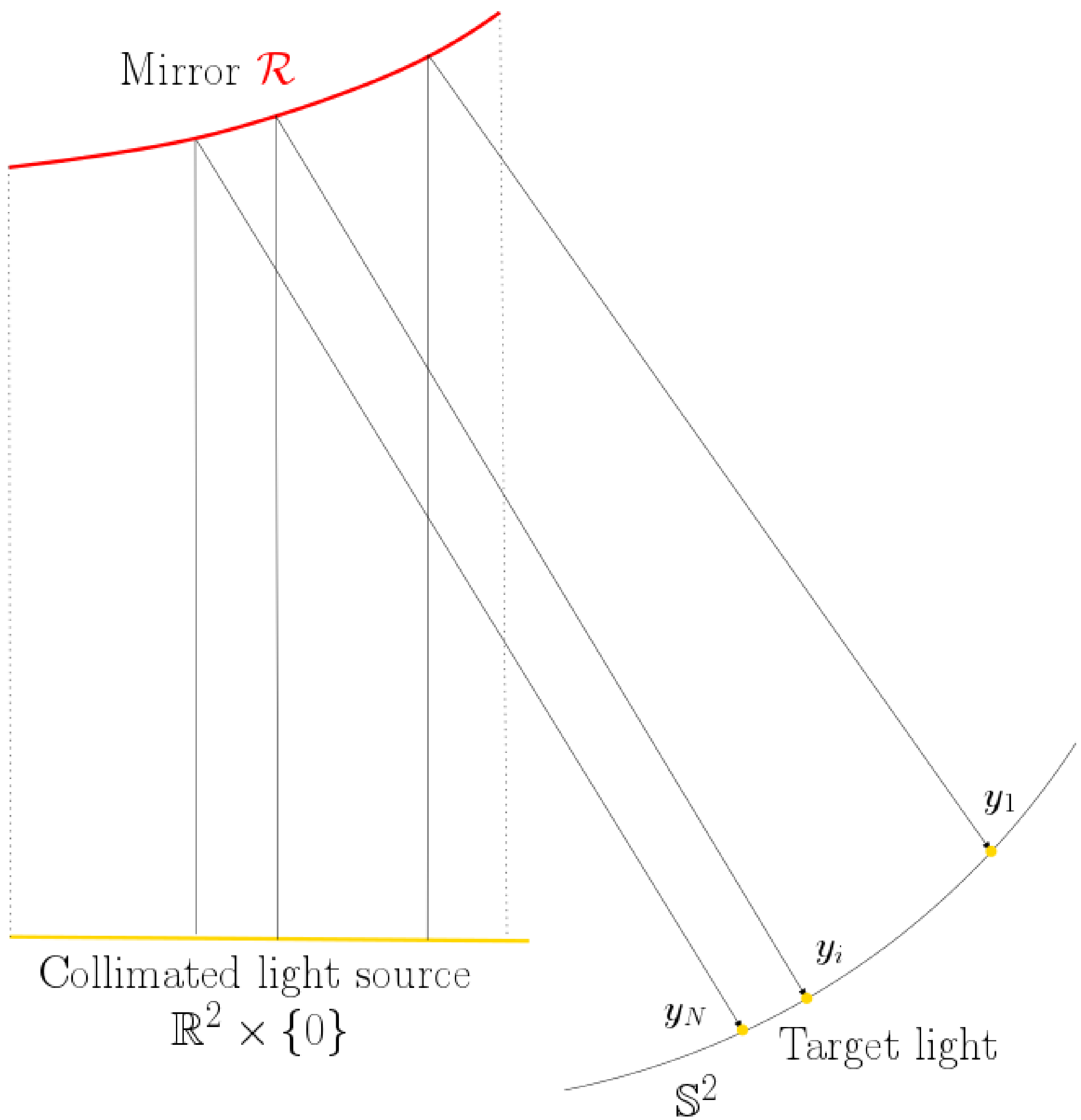






Setting 1: Collimated Source Mirror

Setting: $X = \mathbb{R}^2 \times \{0\}$ and $Y \subseteq \mathbb{S}^2_-$



(Convex) parametrization of $\mathcal{R}$:

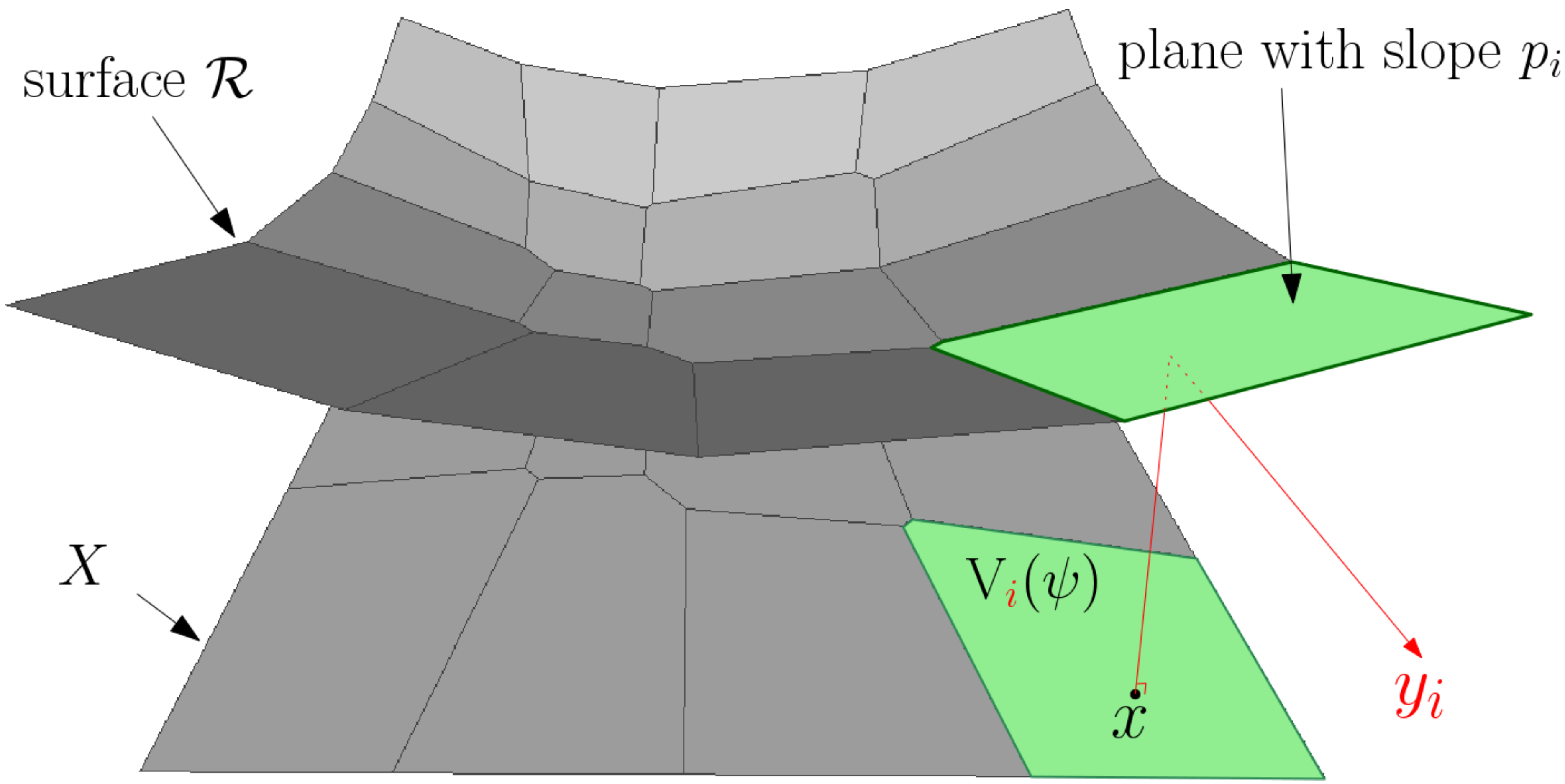
$$\mathcal{R}_\psi : x \in \mathbb{R}^2 \mapsto (x, \max_{1 \leq i \leq N} \langle x | p_i \rangle - \psi_i)$$

where

- $p_i \in \mathbb{R}^2$ = slope of the plane that reflects the ray $(0, 0, 1)$ towards y_i,
- $\psi_i \in \mathbb{R}$ its elevation

Setting 1: Collimated Source Mirror

Visibility cell of y_i: $V_i(\psi) = \{x \in \mathbb{R}^2 \times \{0\} \mid x$ reflected towards direction $y_i\}$



- We set $G_i(\psi) = \int_{V_i(\psi)} \rho(x)dx$ and $G(\psi) = (G_i(\psi))_{1 \le i \le N}$

Collimated Source Mirror problem

Find $\psi \in \mathbb{R}^N$ such that $G(\psi) = \sigma$

Setting 1: Collimated Source Mirror

Computation of $V_i(\psi)$

We recall $\mathcal{R}_\psi(x) = (x, \max_i \langle x|p_i\rangle - \psi_i)$ and we have

$$\begin{aligned} V_i(\psi) &= \{x \in \mathbb{R}^2 \mid \forall j, -\langle x|p_i\rangle + \psi_i \leq -\langle x|p_j\rangle + \psi_j\} \\ &= \{x \in \mathbb{R}^2 \mid \forall j, c(x, y_i) + \psi_i \leq c(x, y_j) + \psi_j\} \\ &=: \mathrm{Lag}_i(\psi) \text{ for } c(x, y) = -\langle x|y\rangle \end{aligned}$$

and

$$\boxed{V_i(\psi) = (\mathbb{R}^2 \times \{0\}) \cap \mathrm{Pow}_i(P)}$$

where $p_i = \frac{p_{\mathbb{R}^2}(y_i - e_z)}{\langle y_i - e_z | e_z\rangle}$ and $\omega_i = 2\psi_i - \|p_i\|^2$

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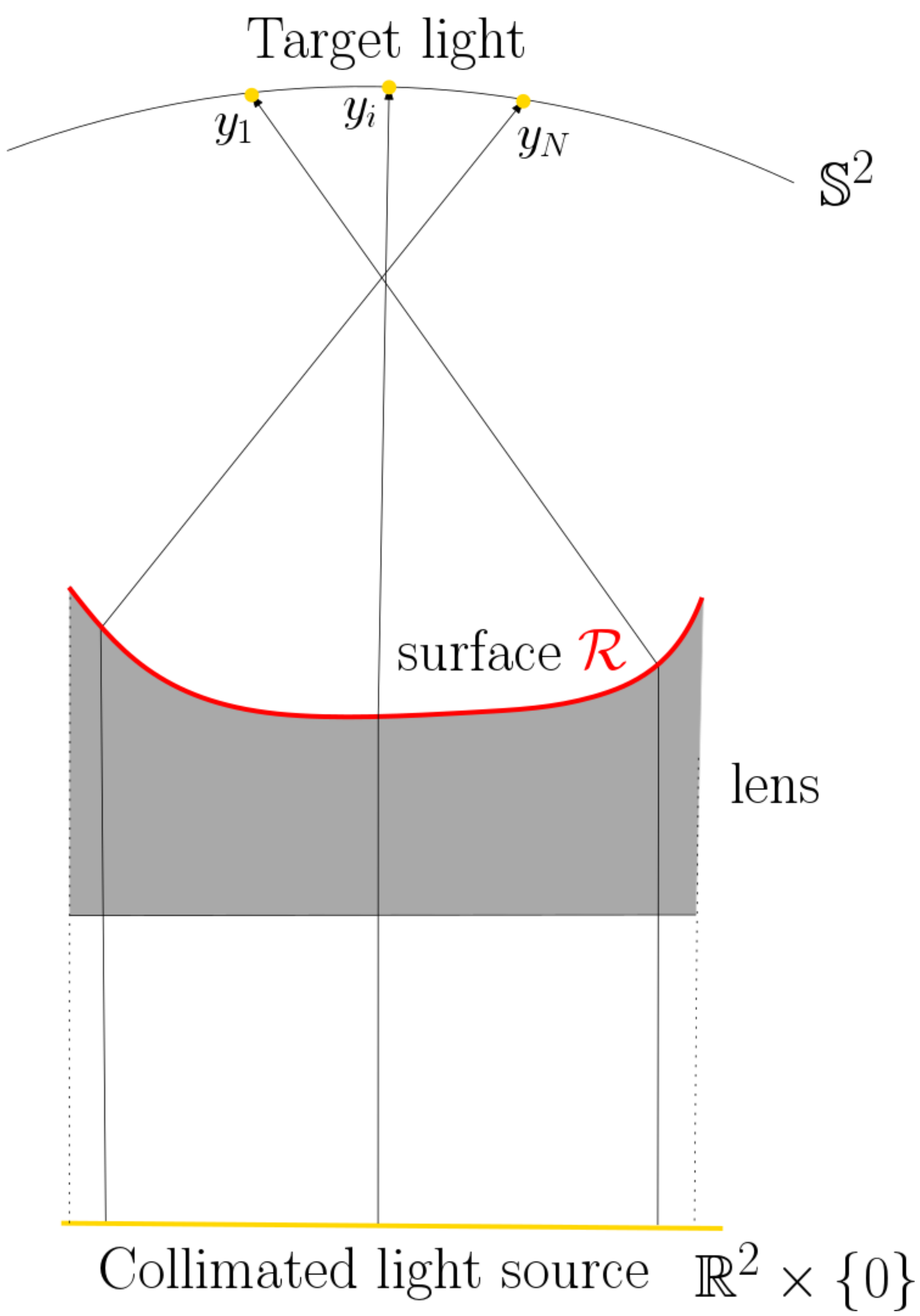
Remark: concave parametrization

$\mathcal{R}_\psi(x) = (x, \min_i \langle x|p_i\rangle + \psi_i)$ and one can replace p_i by its opposite in the previous expressions

Setting 2: Collimated Source Lens

Setting: $X = \mathbb{R}^2 \times \{0\}$, $Y \subseteq \mathbb{S}^2_+$ and κ = ratio of the refractive indices



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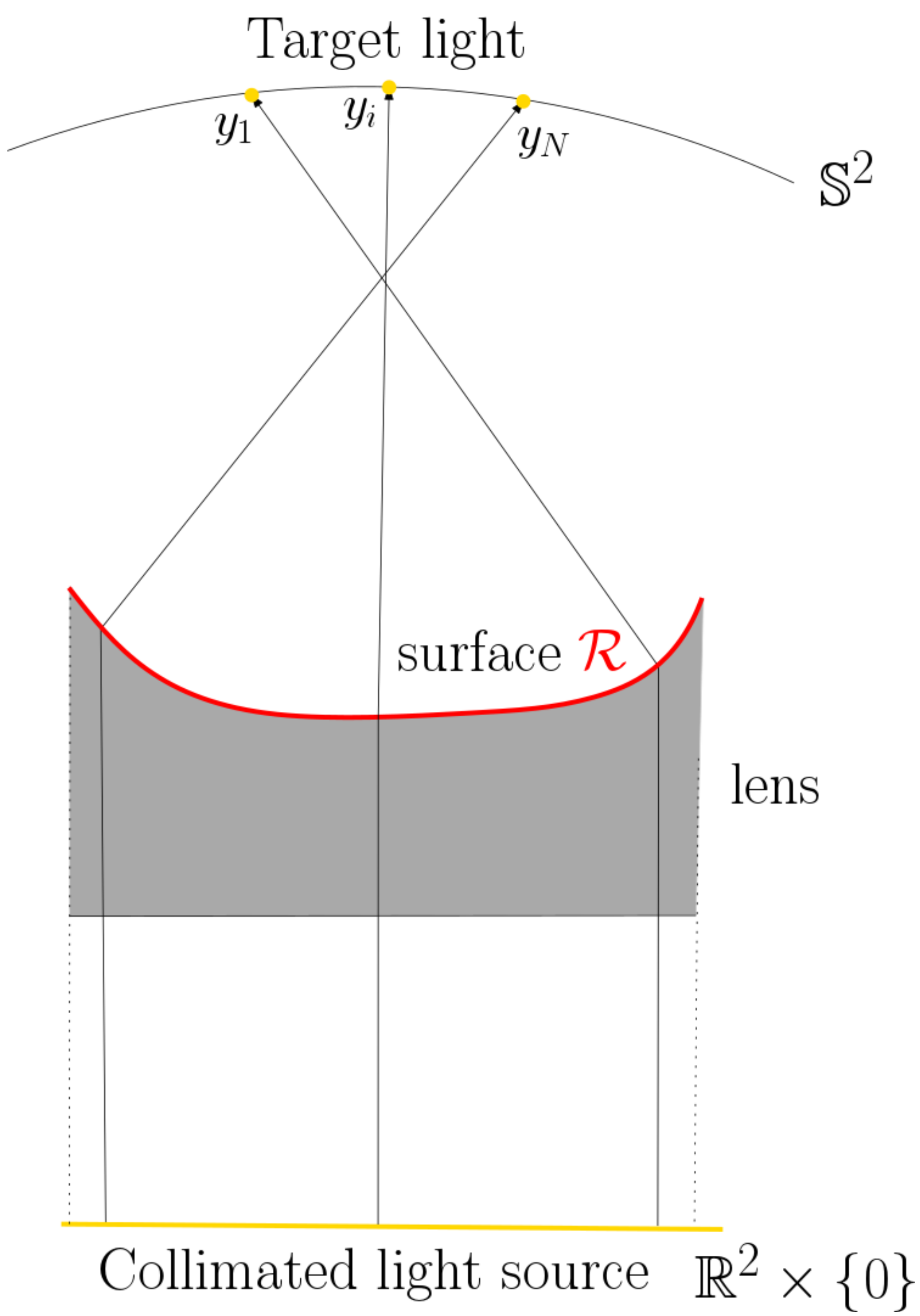

Convex parametrization

$$\mathcal{R}_\psi(x) = (x, \max_{1 \leq i \leq N} \langle x | p_i \rangle - \psi_i)$$

and

$$V_i(\psi) = \{x \in \mathbb{R}^2 \times \{0\} \mid \text{ x } \textit{refracted} \text{ towards } y_i\}$$
$$=: \mathrm{Lag}_i(\psi) \text{ for } c(x, y_i) = -\langle x | p_i \rangle$$

$$V_i(\psi) = (\mathbb{R}^2 \times \{0\}) \cap \mathrm{Pow}_i(P)$$

where $p_i = -\frac{p_{\mathbb{R}^2}(y_i - \kappa e_z)}{\langle y_i - \kappa e_z | e_z \rangle}$ and $\omega_i = 2\psi_i - \|p_i\|^2$

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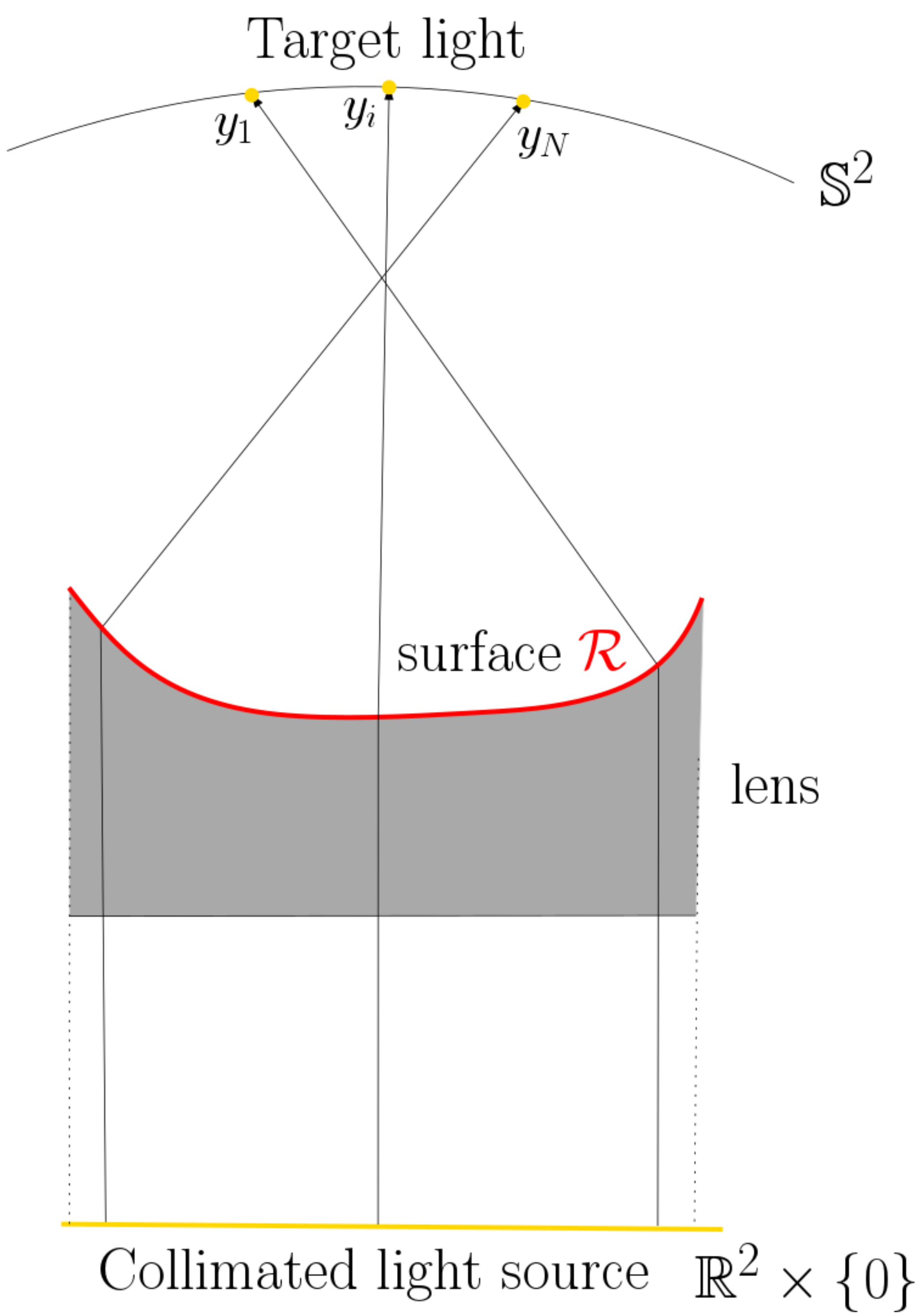

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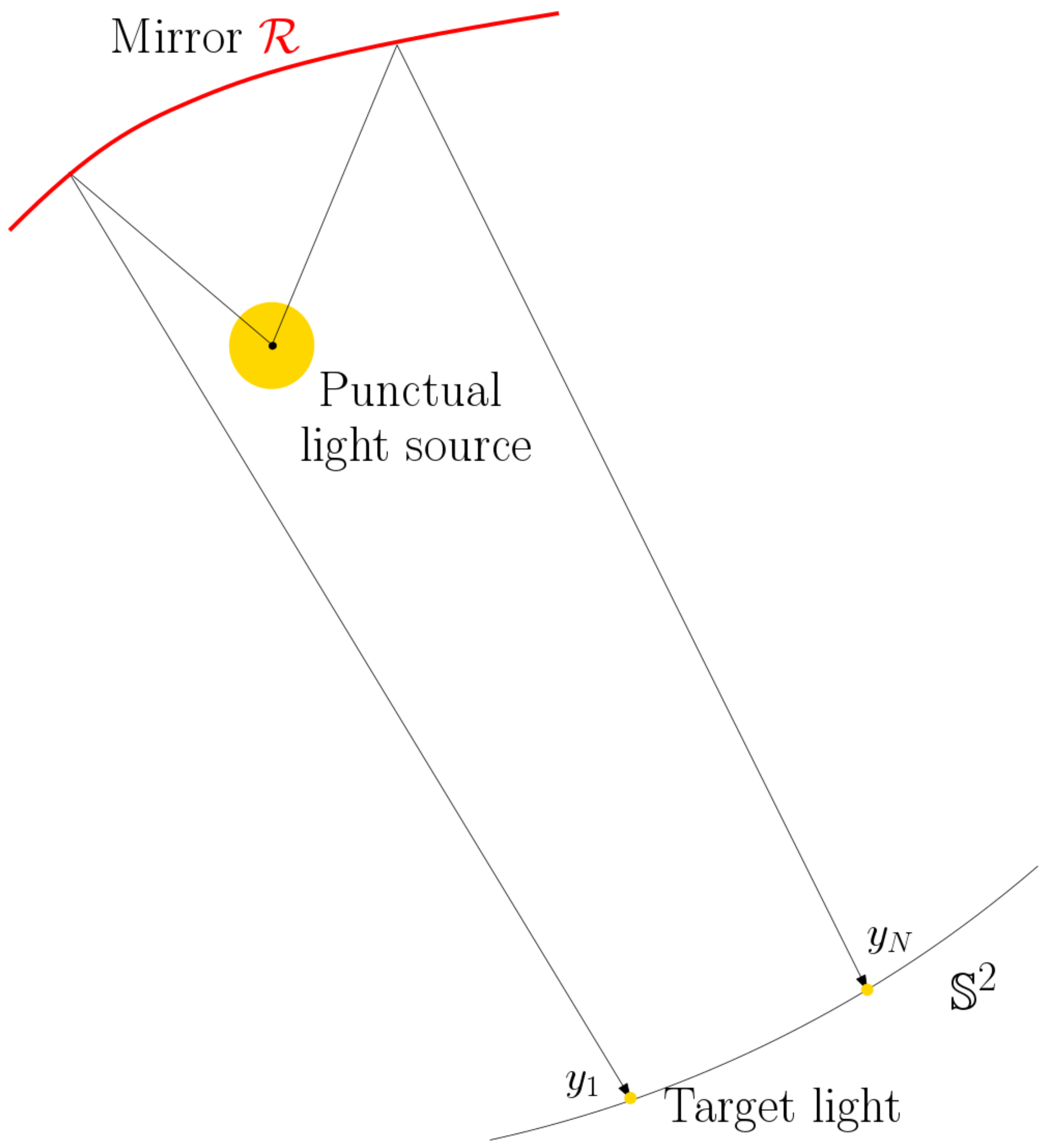
Remark: concave parametrization

$$\mathcal{R}_\psi(x) = (x, \min_i \langle x | p_i \rangle + \psi_i)$$

Setting 3: Point Source Mirror

Setting: $X = \mathbb{S}^2$ and $Y \subseteq \mathbb{S}^2_-$



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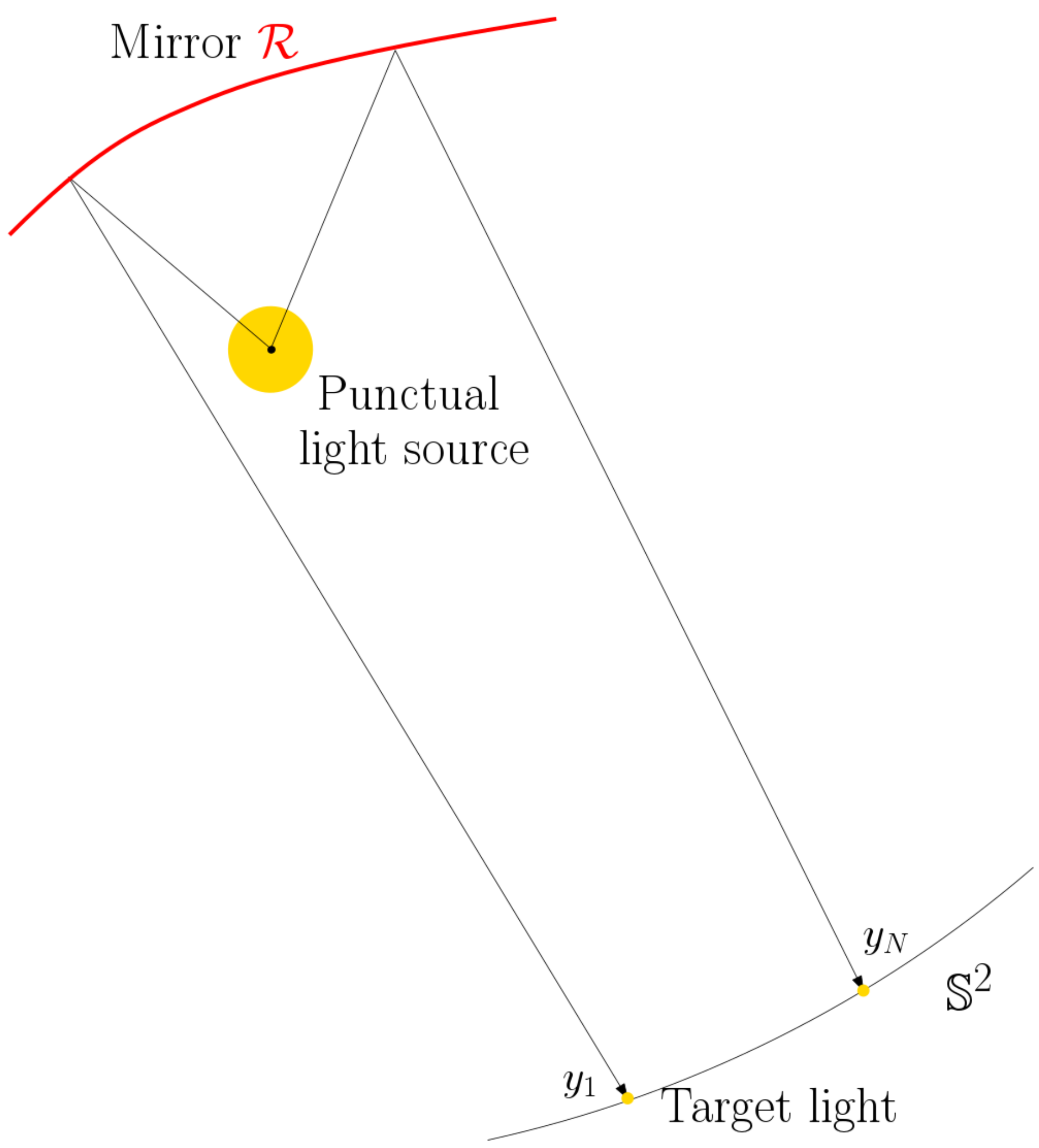

Parametrization: intersection of confocal paraboloids $\implies$ convex

$$\mathcal{R}_\psi : x \in \mathbb{S}^2 \mapsto \max_{1 \leq i \leq N} \frac{\psi_i}{1 - \langle x | y_i \rangle}$$

where ψ_i is the focal distance of the i-th paraboloid, and we have

$$V_i(\psi) = \mathbb{S}^2 \cap \mathrm{Pow}_i(P)$$

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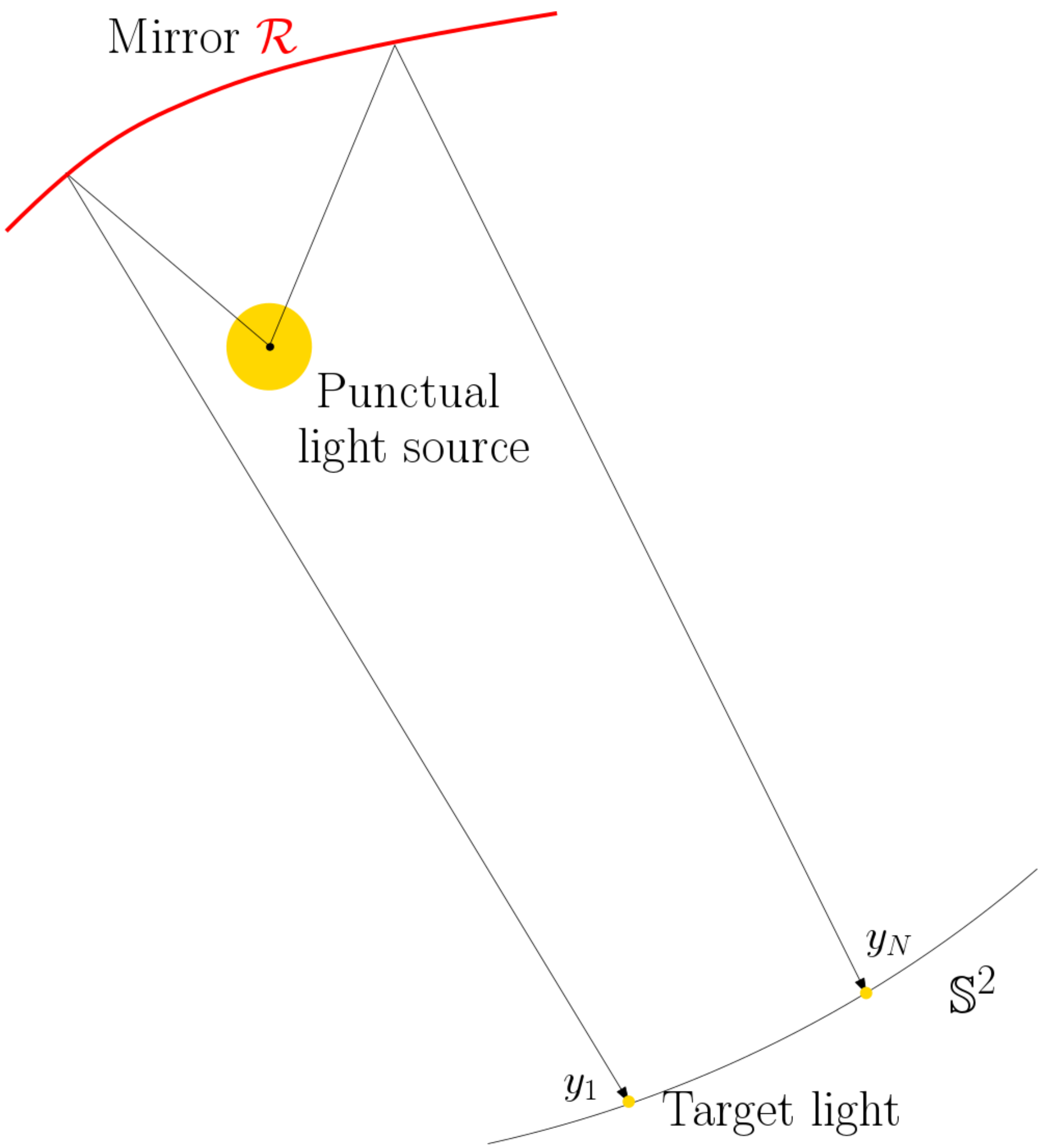

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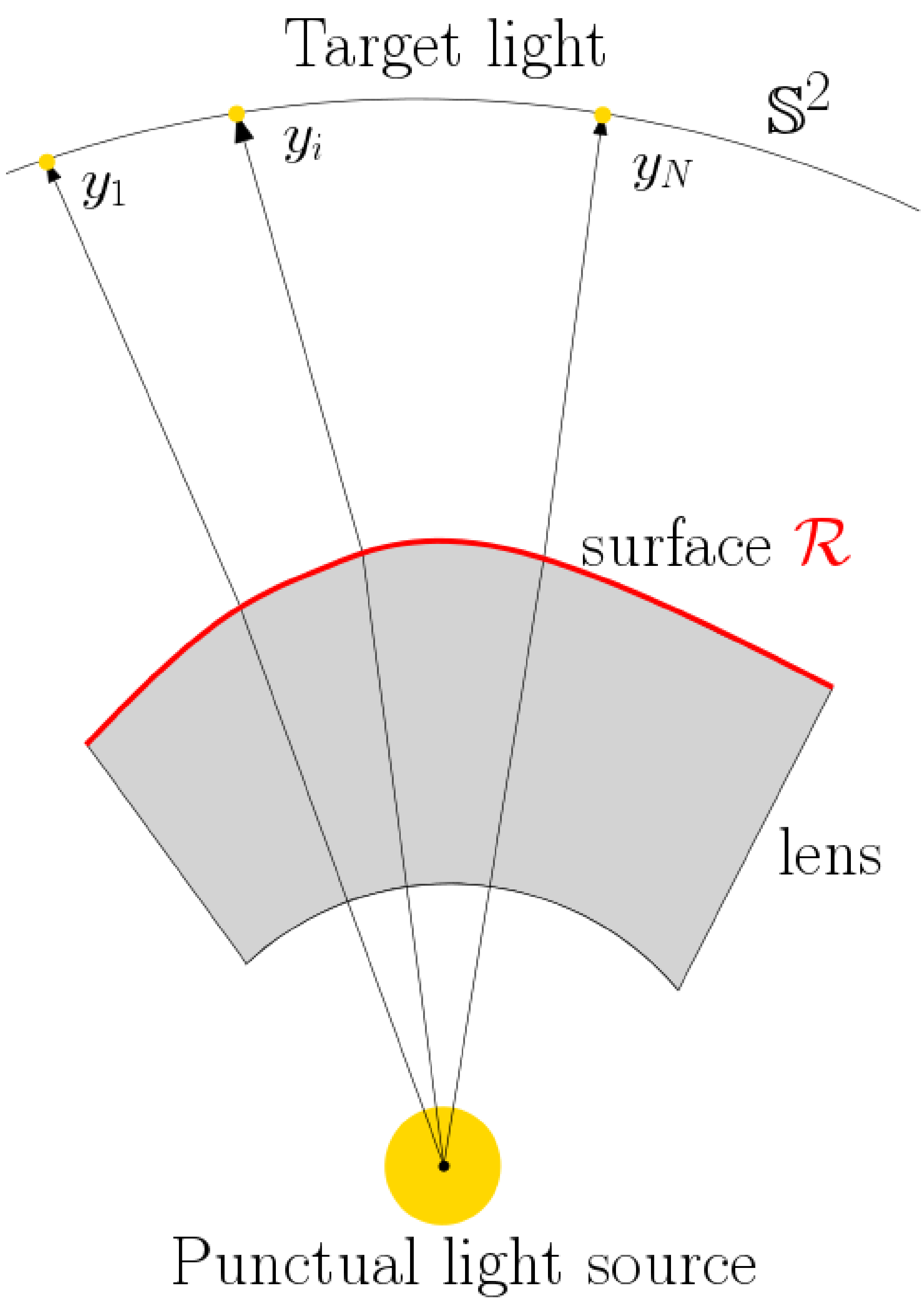
Remark: union of confocal paraboloids

Setting 4: Point Source Lens

Setting: $X = \mathbb{S}^2$, $Y \subseteq \mathbb{S}^2_+$ and $\kappa < 1$



Parametrization: intersection of confocal ellipsoids $\implies$ convex

$$\mathcal{R}_\psi : x \in \mathbb{S}^2 \mapsto \max_{1 \leq i \leq N} \frac{\psi_i}{1 - \kappa \langle x | y_i \rangle}$$

where ψ_i is one of the focal distances of the i-th ellipsoid, and

$$V_i(\psi) = \mathbb{S}^2 \cap \mathrm{Pow}_i(P)$$

Remark: union of confocal elipsoids

Common structure

Semi discrete Monge Ampère equation

Find $\psi \in \mathbb{R}^N$ such that $G(\psi) = \sigma$

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Efficient evaluation of G

- Visibility cells: $V_i(\psi) = X \cap \mathrm{Pow}_i(P)$
- Collimated: $X = \mathbb{R}^2 \times \{0\}$
- Punctual: $X = \mathbb{S}^2$

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Damped Newton algorithm: convergence results known for

- Quadratic cost in the plane [Mirebeau, 2015]
- Many cost functions (MTW) [Kitagawa et al., 2016]

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Damped Newton algorithm: convergence results known for

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$\Longrightarrow$ We will use the same algorithm

Generic algorithm

Overview

Algorithm: Mirror / lens construction

Input A light source intensity function ρ_{in}
A target light intensity function σ_{in}
A tolerance $\eta > 0$

Output A triangulation $\mathcal{R}_T$ of a mirror or lens $\mathcal{R}$

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Step 1 Initialization
$T, \rho \leftarrow \texttt{DISCRETIZATION\_SOURCE}(\rho_{in})$
$Y, \sigma \leftarrow \texttt{DISCRETIZATION\_TARGET}(\sigma_{in})$
$\psi^0 \leftarrow \texttt{INITIAL\_WEIGHTS}(Y)$

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Step 2 Solve $G(\psi) = \sigma$

$\psi \leftarrow \texttt{DAMPED\_NEWTON}(T, \rho, Y, \sigma, \psi^0, \eta)$

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Step 3 Construct a triangulation $\mathcal{R}_T$ of $\mathcal{R}$

$\mathcal{R}_T \leftarrow \texttt{SURFACE\_CONSTRUCTION}(\psi, \mathcal{R}_\psi)$

Step 1: Initialization

Discretization:

- ρ_{in} is discretized by a piecewise affine function ρ on a *triangulation* T
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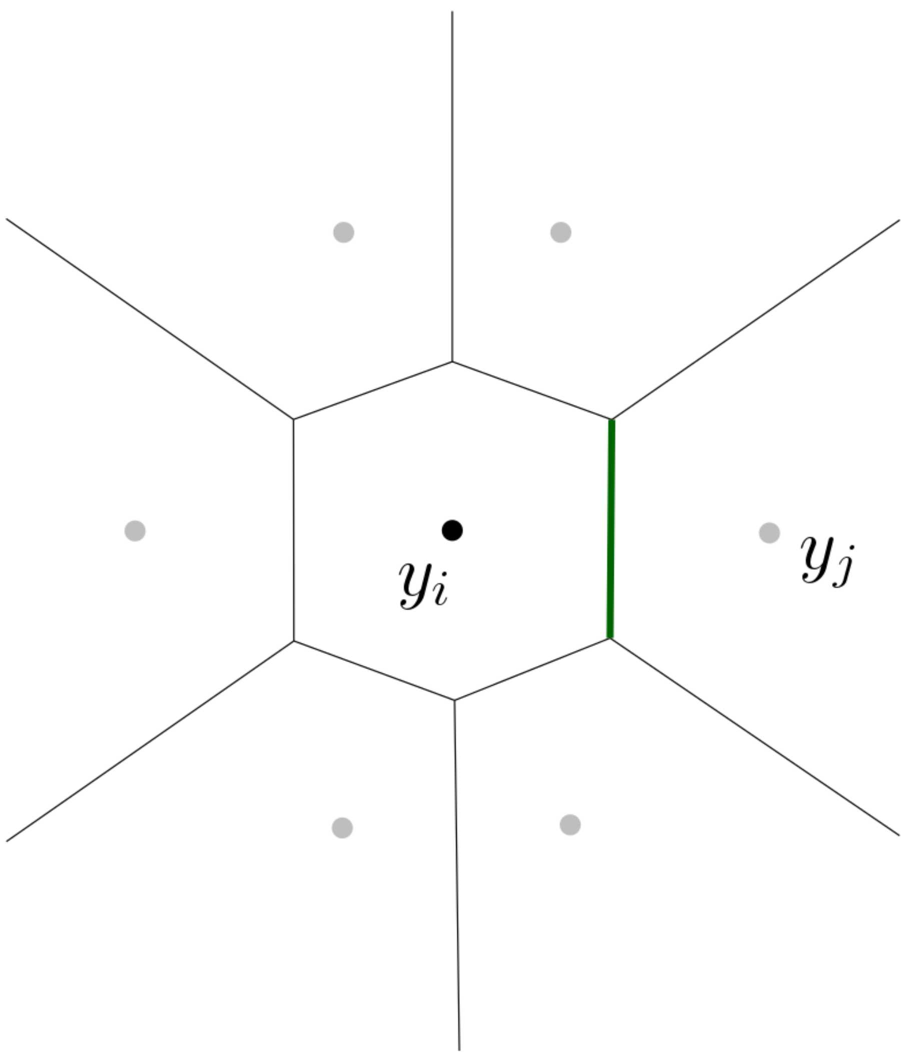
Initial weights: we must ensure that at each step $\forall i, G_i(\psi) > 0$



$$\frac{\partial G_i}{\partial \psi_j}(\psi) \propto \int_{V_{i,j}(\psi)} \rho(x)dx$$

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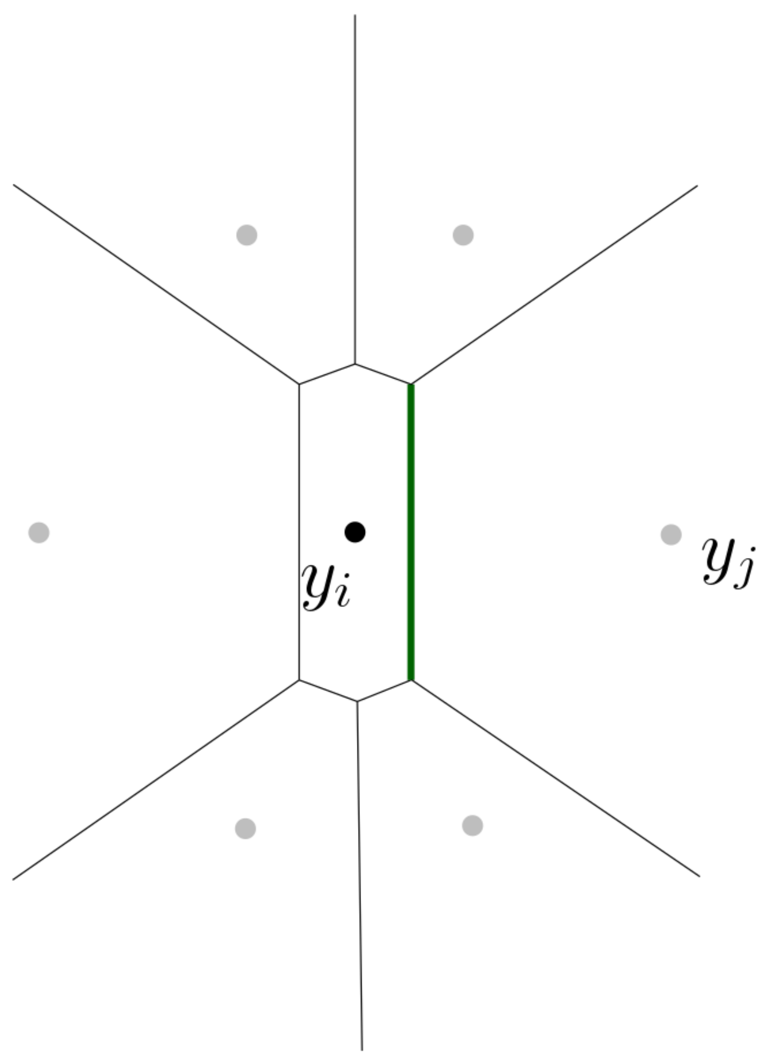

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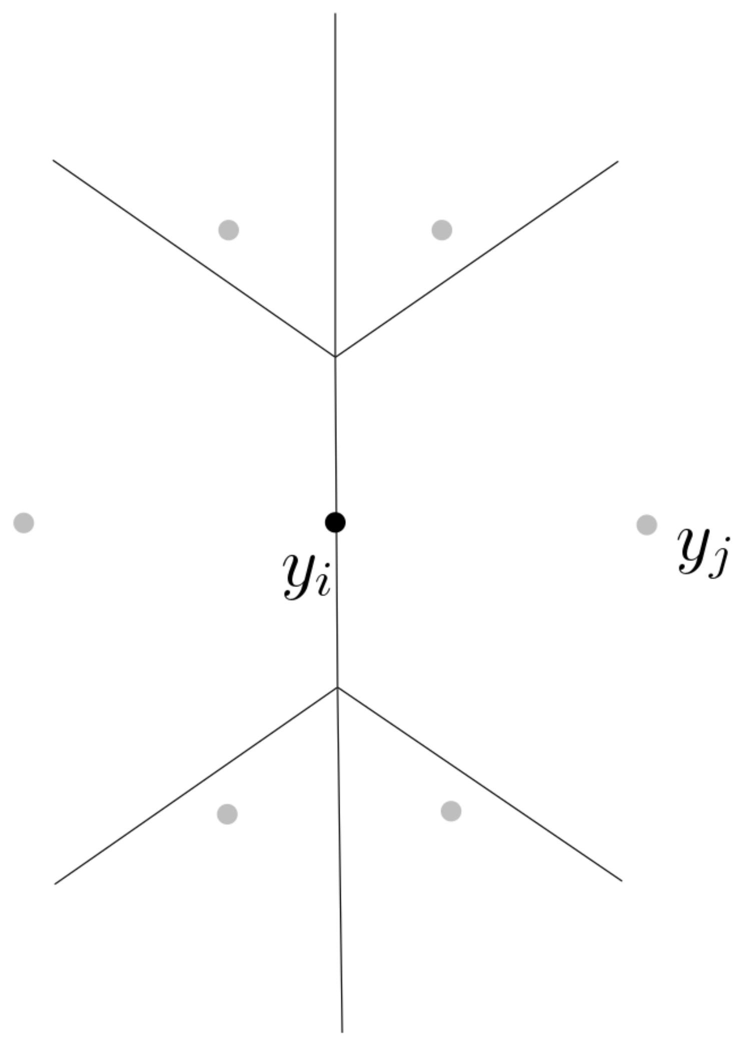


$$\frac{\partial G_i}{\partial \psi_j}(\psi) \propto \int_{V_{i,j}(\psi)} \rho(x)dx$$

$\implies \frac{\partial G_i}{\partial \psi_j}$ is not continuous!

Step 1: Initialization

Discretization:

- ρ_{in} is discretized by a piecewise affine function ρ on a *triangulation* T
- σ_{in} is discretized by a discrete measure σ on a 3D point cloud Y

Initial weights: we must ensure that at each step $\forall i, G_i(\psi) > 0$

At the beginning:

- simple settings $\implies$ easy choices can be made
- more complex configurations like *pillows* $\implies$ other strategies

Step 2: Damped Newton algorithm

Algorithm: Mirror / lens construction

Input A function $\rho : T \to \mathbb{R}^+$

A discrete measure $\sigma = (\sigma_i)_{1 \leq i \leq N}$ supported on Y

An initial vector of weights ψ^0

A tolerance $\eta > 0$

Output A vector $\psi \in \mathbb{R}^N$

Step 1 Transformation into an optimal transport problem

If $X = \mathbb{R}^2 \times \{0\}$, then $\tilde{\psi}^0 = \psi^0$ and $\tilde{G}_i = G_i$

If $X = \mathbb{S}^2$, then $\tilde{\psi}^0 = \ln(\psi^0)$ and $\tilde{G}_i = G_i \circ \exp$

Step 2: Damped Newton algorithm

Algorithm: Mirror / lens construction

Step 2 Solve $\tilde{G}(\tilde{\psi}) = \sigma$

Initialization $\epsilon_0 = \min[\min_i G_i(\psi^0), \min_i \sigma_i]$

$k = 0$

While $\|\tilde{G}(\tilde{\psi}^k) - \sigma\|_\infty > \eta$

- Compute $d_k = -D\tilde{G}(\tilde{\psi}^k)^+(\tilde{G}(\tilde{\psi}^k) - \sigma)$

- Determine the minimum $\ell \in \mathbb{N}$ such that $\tilde{\psi}^{k,l} := \tilde{\psi}^k + 2^{-\ell} d_k$ satisfies:

$$\begin{cases} \min_i \tilde{G}_i(\tilde{\psi}^{k,l}) \geq \epsilon_0 \\ \|\tilde{G}(\tilde{\psi}^{k,l}) - \sigma\|_\infty \leq (1 - 2^{-(\ell+1)})\|\tilde{G}(\tilde{\psi}^k) - \sigma\|_\infty \end{cases}$$

- Set $\tilde{\psi}^{k+1} = \tilde{\psi}^k + 2^{-\ell} d_k$ and $k \leftarrow k + 1$

Return $\psi := \tilde{\psi}^k$ if $X = \mathbb{R}^2 \times \{0\}$ or

$\psi := \exp(\tilde{\psi}^k)$ if $X = \mathbb{S}^2$

Step 2: Damped Newton algorithm

Computation of DG: automatic differentiation $\Longrightarrow$ genericity

Step 2: Damped Newton algorithm

Computation of DG: automatic differentiation $\implies$ genericity

Theorem ([Mérigot, M., Thibert, 2017]

Assume ρ is a regular simplicial measure and that the points $p_1, \ldots, p_N$ are in generic position. Then:

- $\tilde{G}$ is of class $\mathcal{C}^1$ on $\mathbb{R}^N$,
- $\tilde{G}$ is stricty monotone in the sense

$$\forall \psi \in K^+, \forall v \in \{cst\}^\perp \setminus \{0\}, \langle D\tilde{G}(\psi) v | v \rangle < 0$$

$\implies$ the proposed damped Newton algorithm converges in a finite number of steps and

$$\|\tilde{G}(\psi^{k+1}) - \sigma\|_\infty \leq \left(1 - \frac{\tau^*}{2}\right) \|\tilde{G}(\psi^k) - \sigma\|_\infty$$

where $\tau^* \in]0, 1]$ depends on ρ, σ and ϵ_0.

Step 2: Damped Newton algorithm

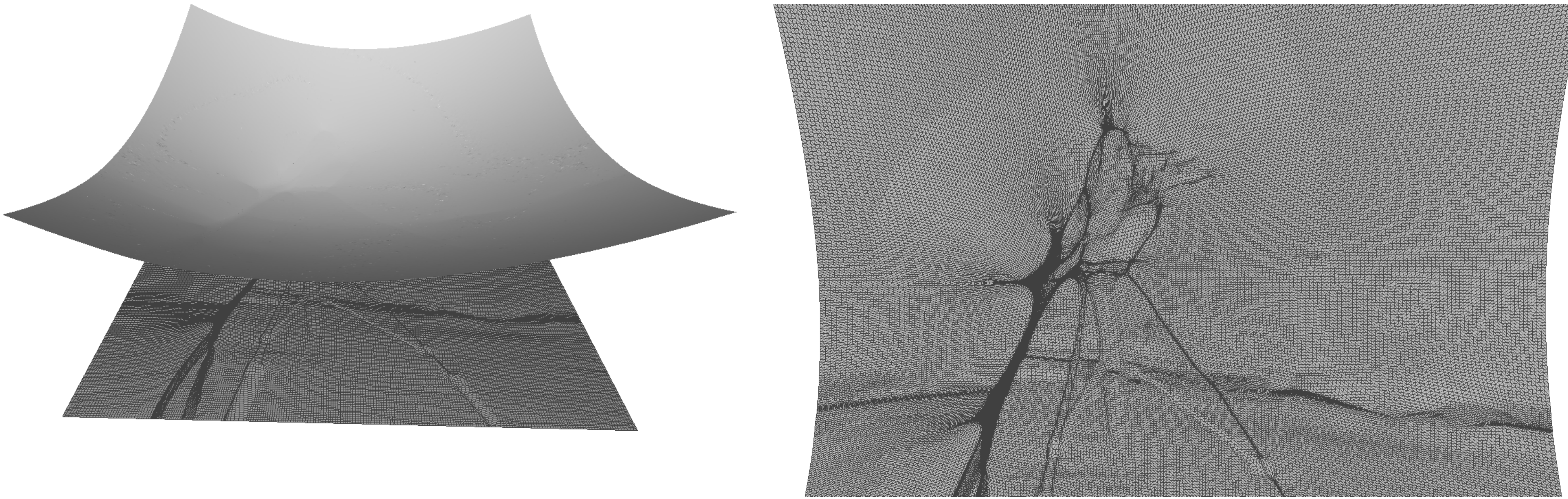
Computation of DG: automatic differentiation $\implies$ genericity

Convergence known for:

- Collimated settings: mirror and lens (convex and concave)
- Punctual settings: mirror (intersection), lens (union)

Step 3: Surface construction

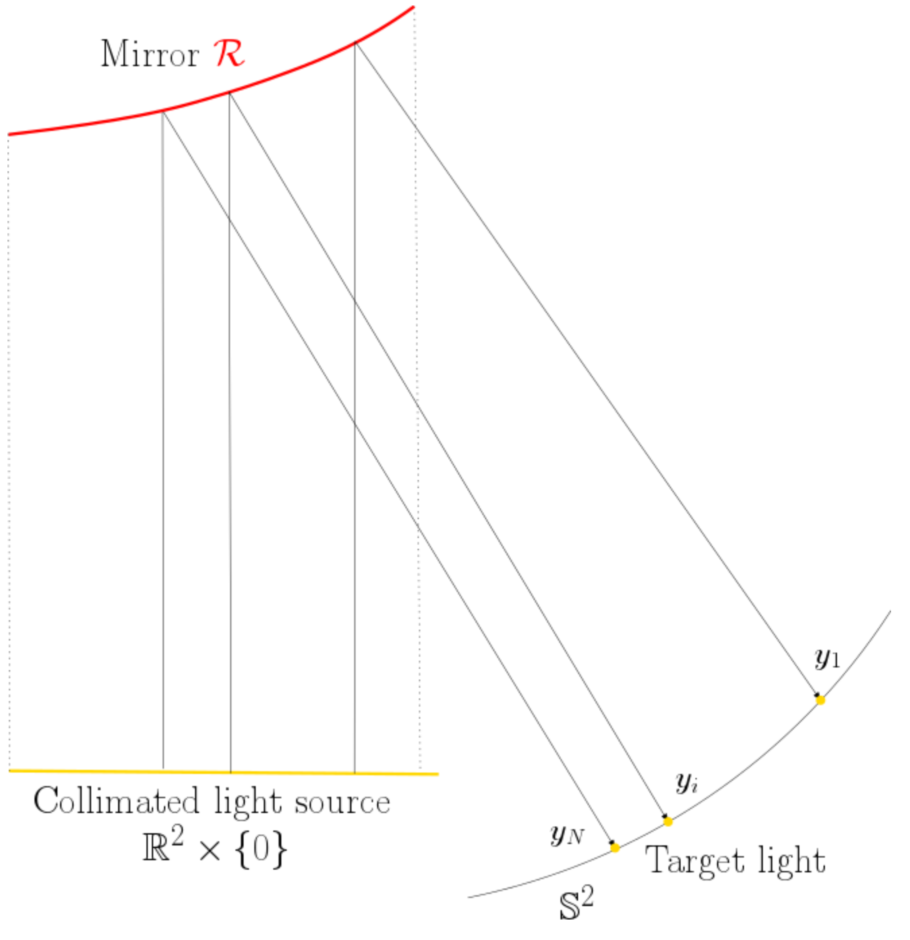
$\mathcal{R}_T$ is the *lifted* triangulation *dual* to the *Visibility* diagram

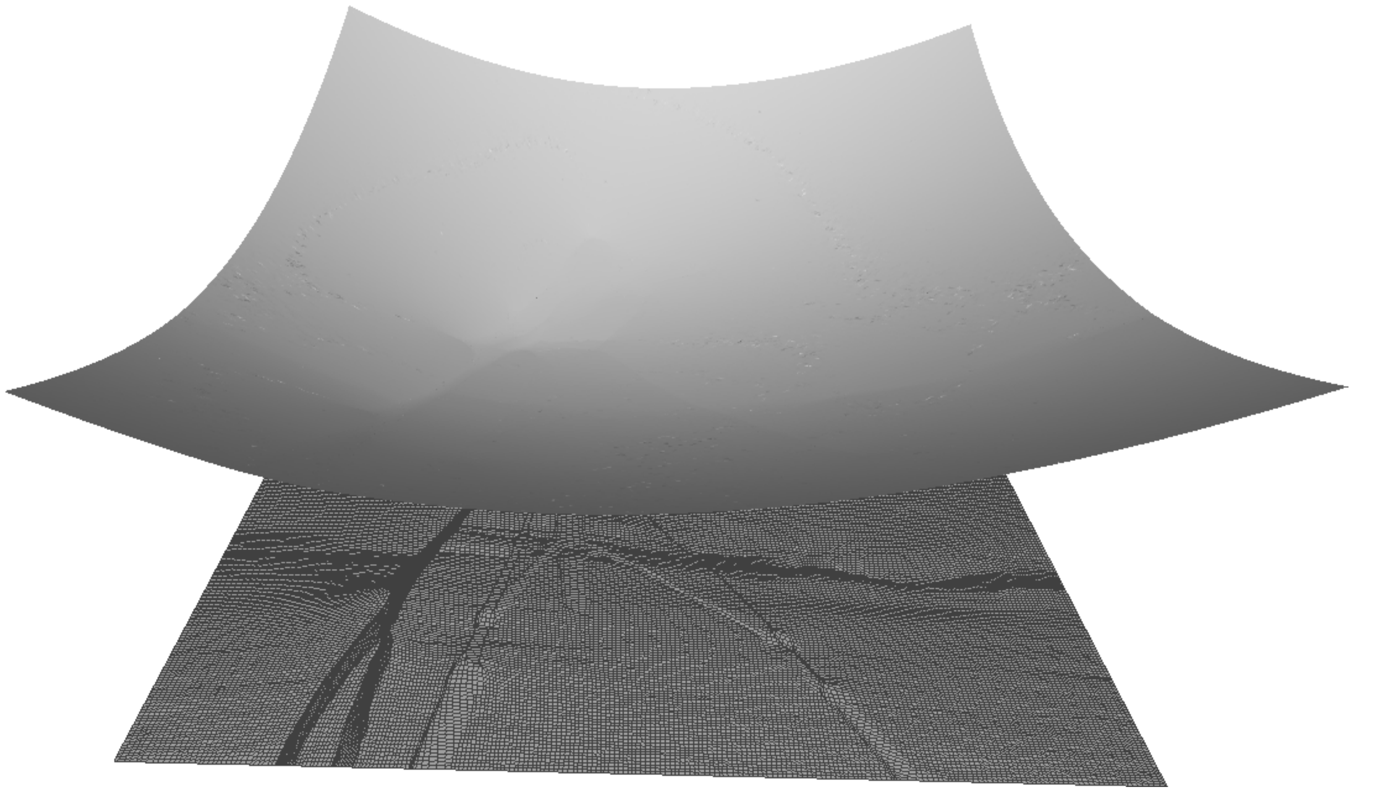


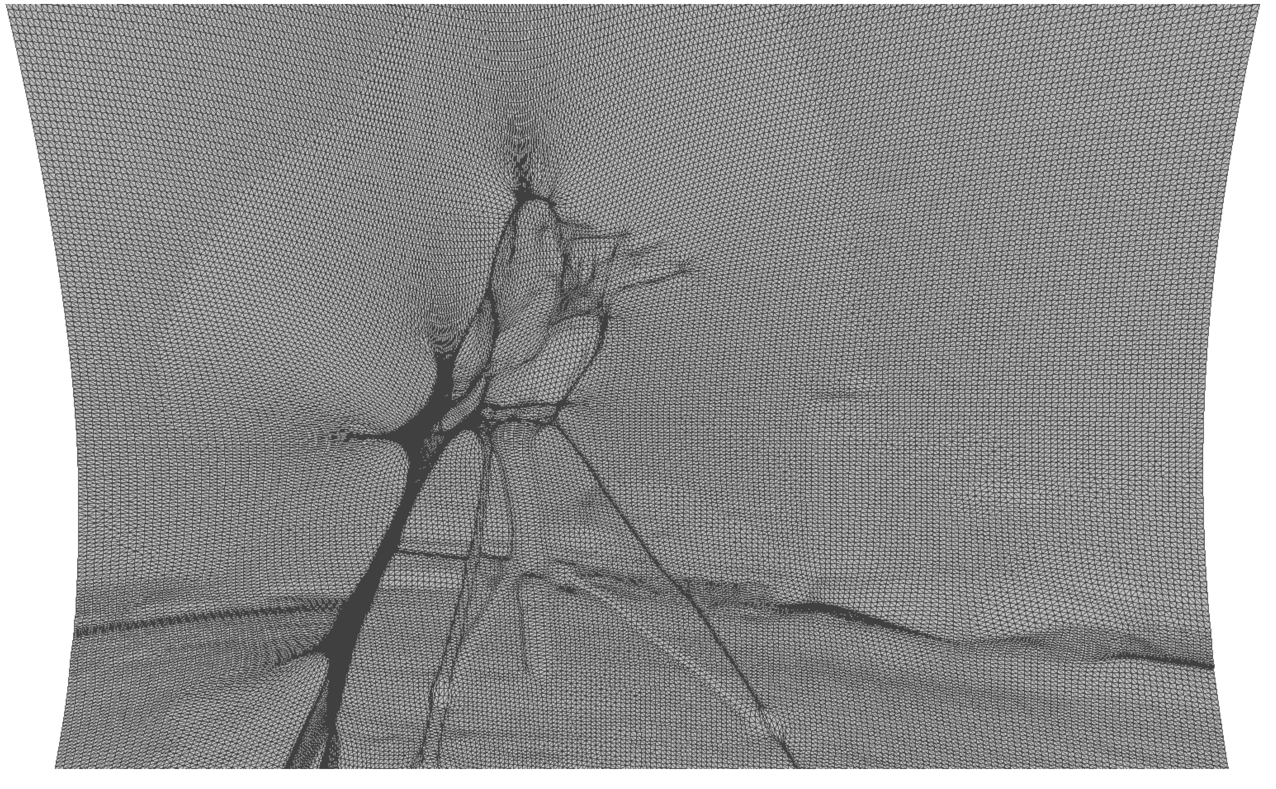
Collimated Source Mirror (convex)

Numerical results

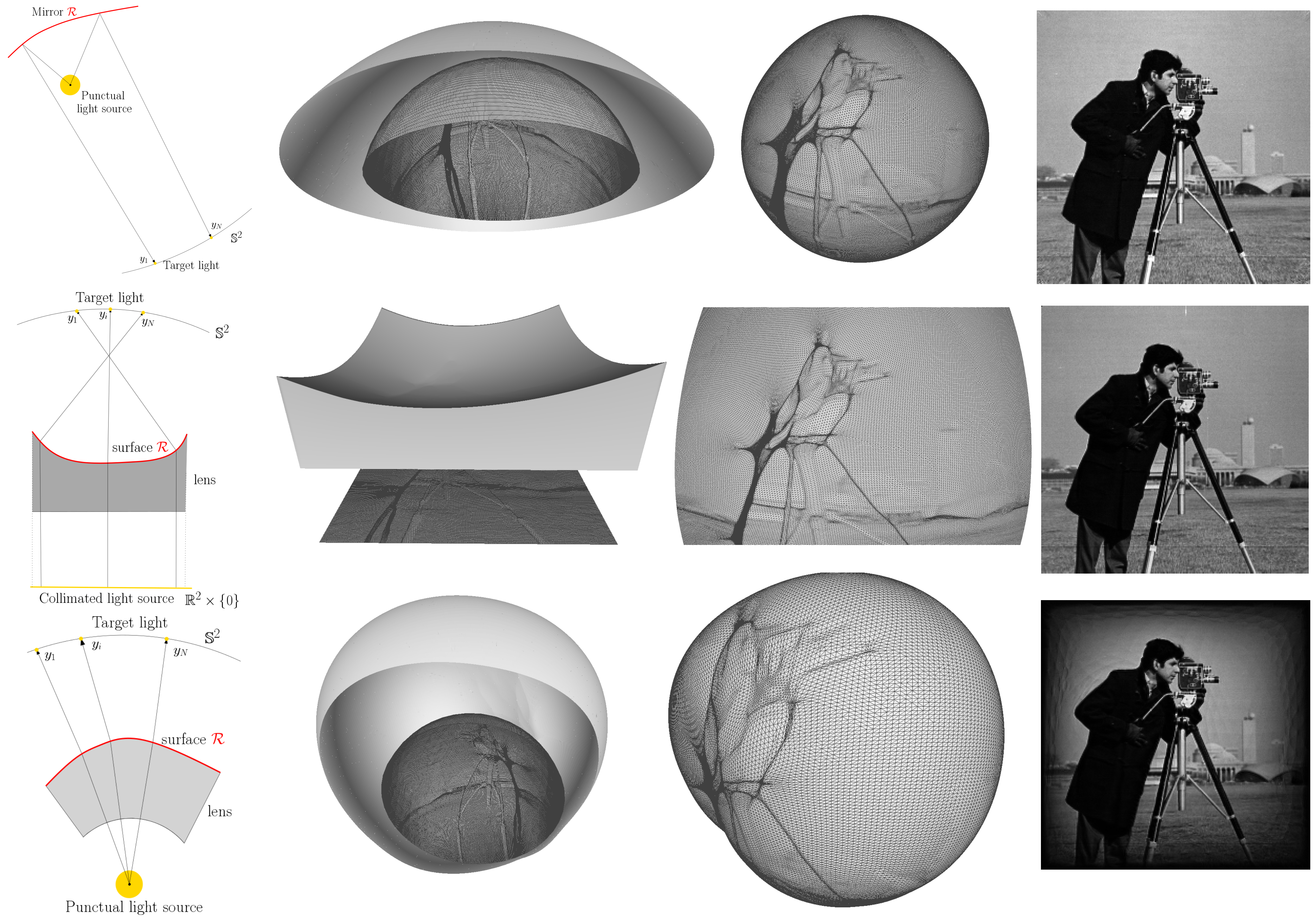
General framework





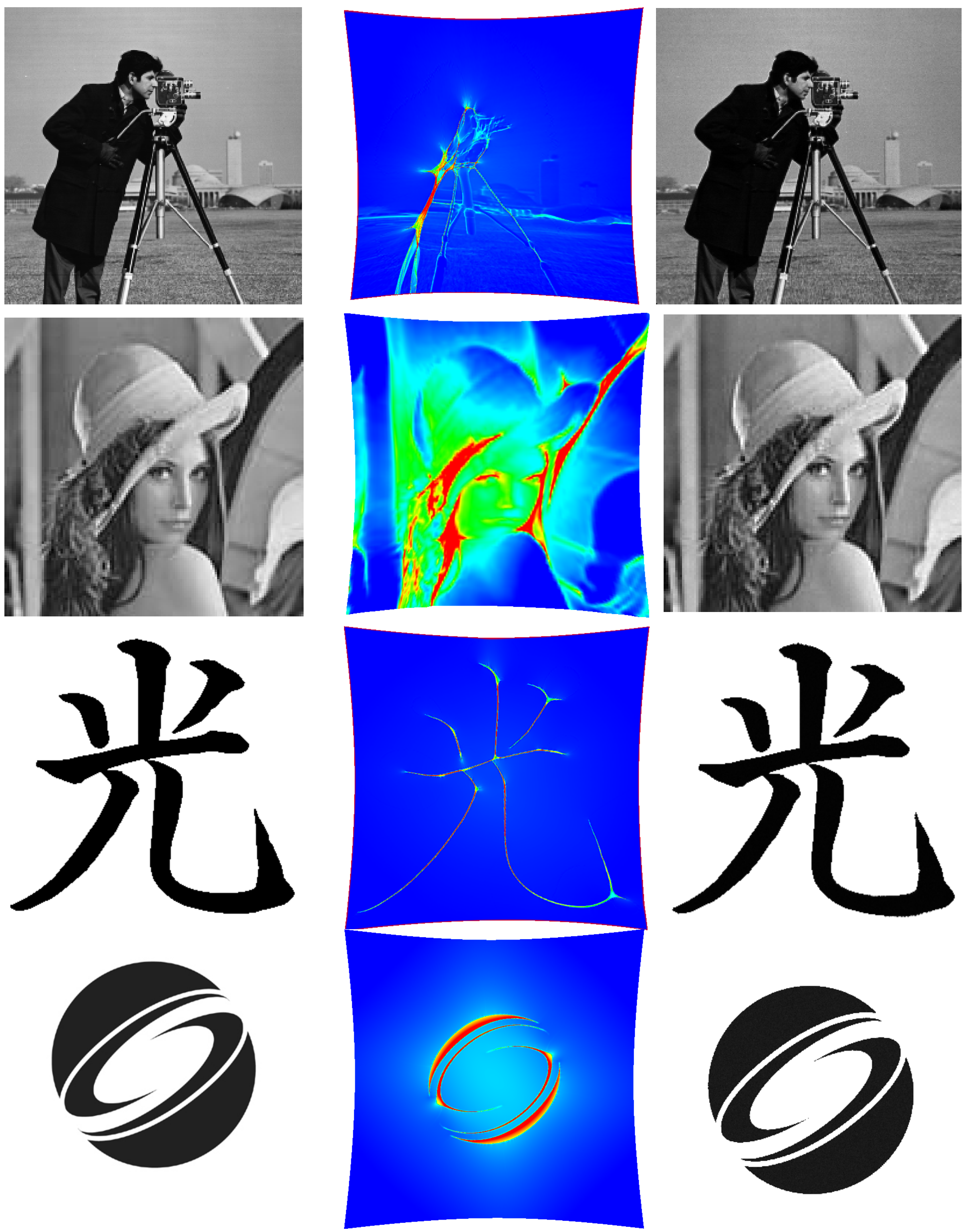


General framework



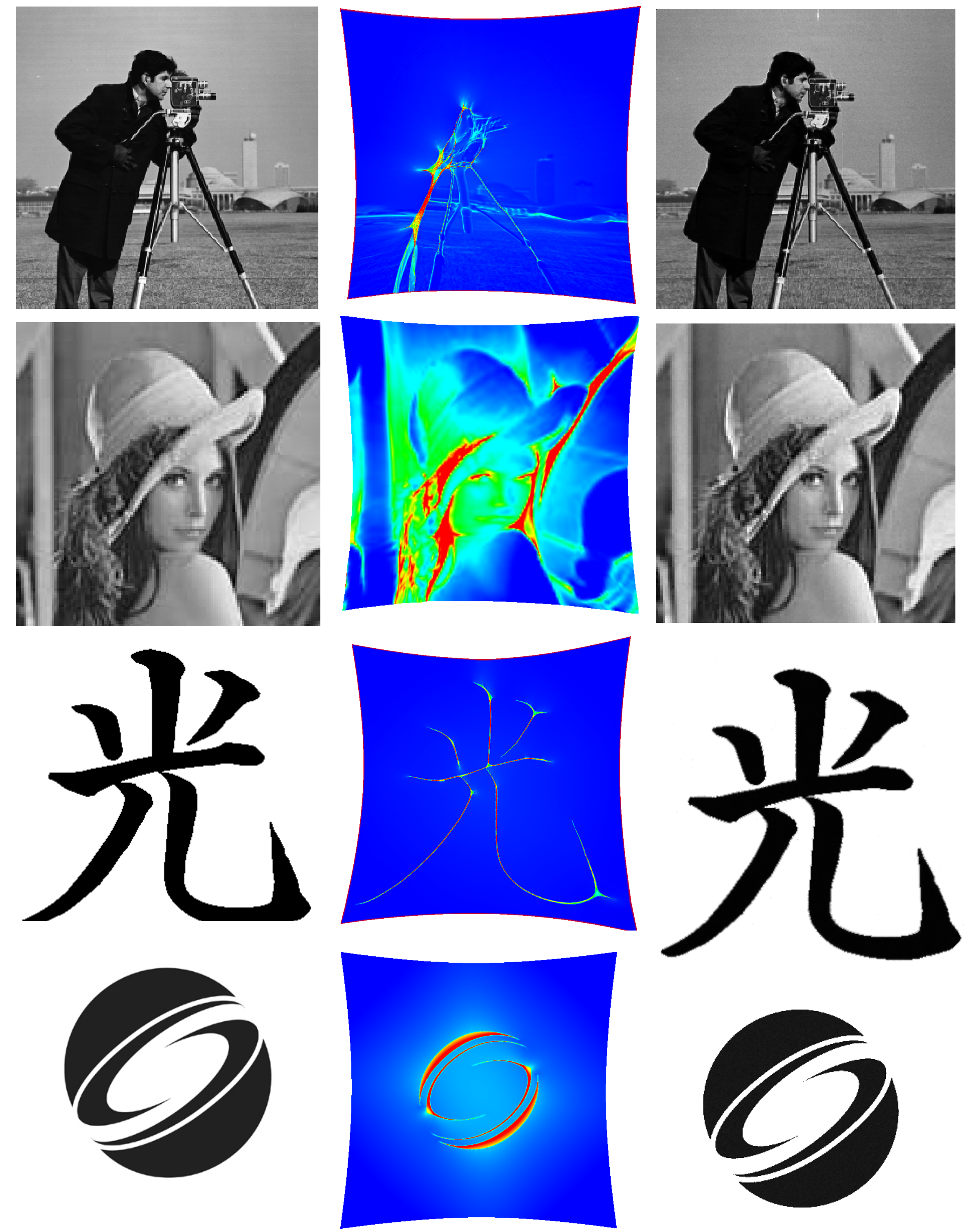
Collimated source

Target / Mean curvature / Forward simulation



Convex Collimated Source Mirror problem with a uniform light source

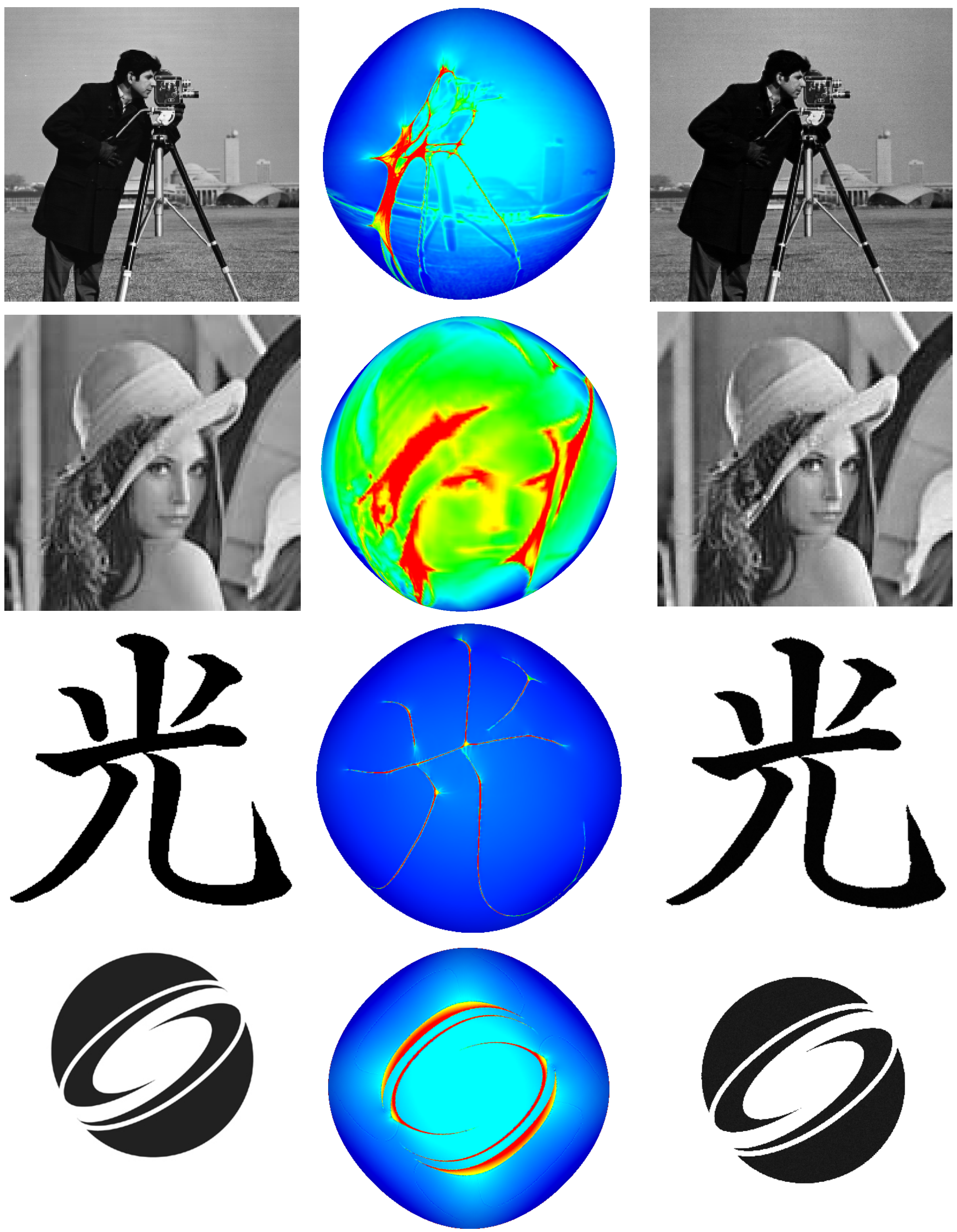
Target / Mean curvature / Forward simulation



Concave Collimated Source Lens problem with a uniform light source

Punctual source

Target / Mean curvature / Forward simulation

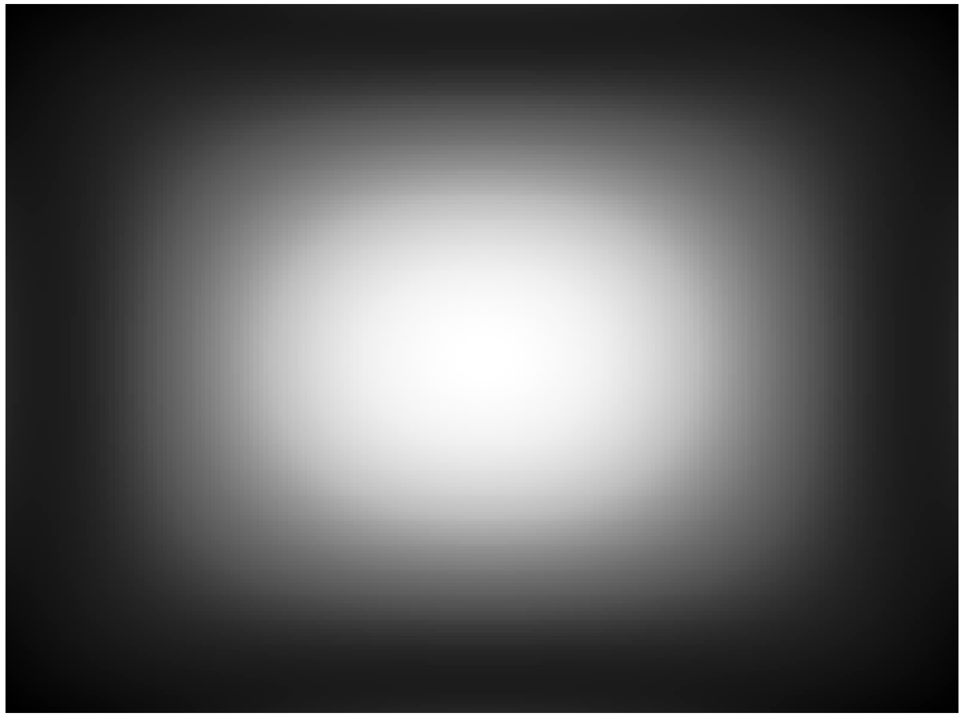


Concave Point Source Mirror problem with a uniform light source

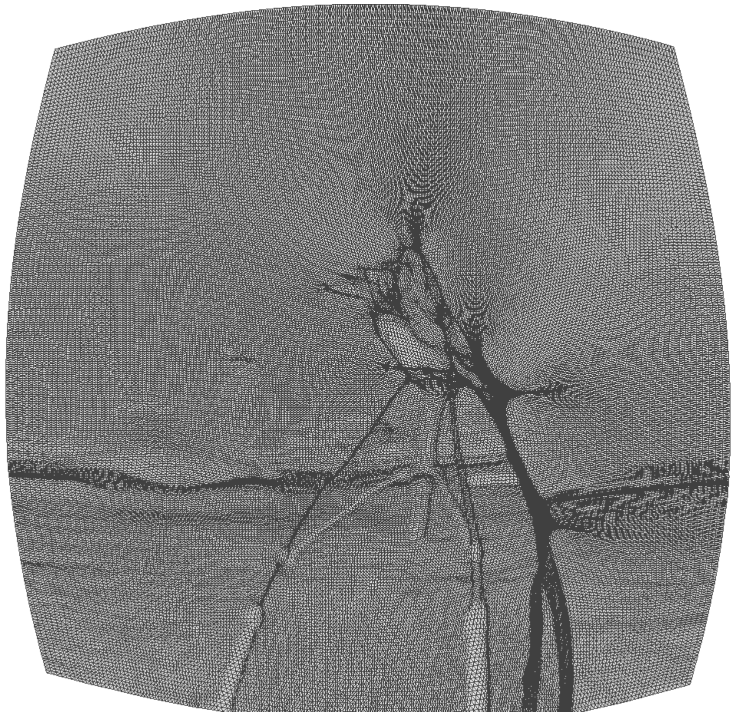
Target / Mean curvature / Forward simulation

Point Source Lens problem with a uniform light source

Non uniform source



Source

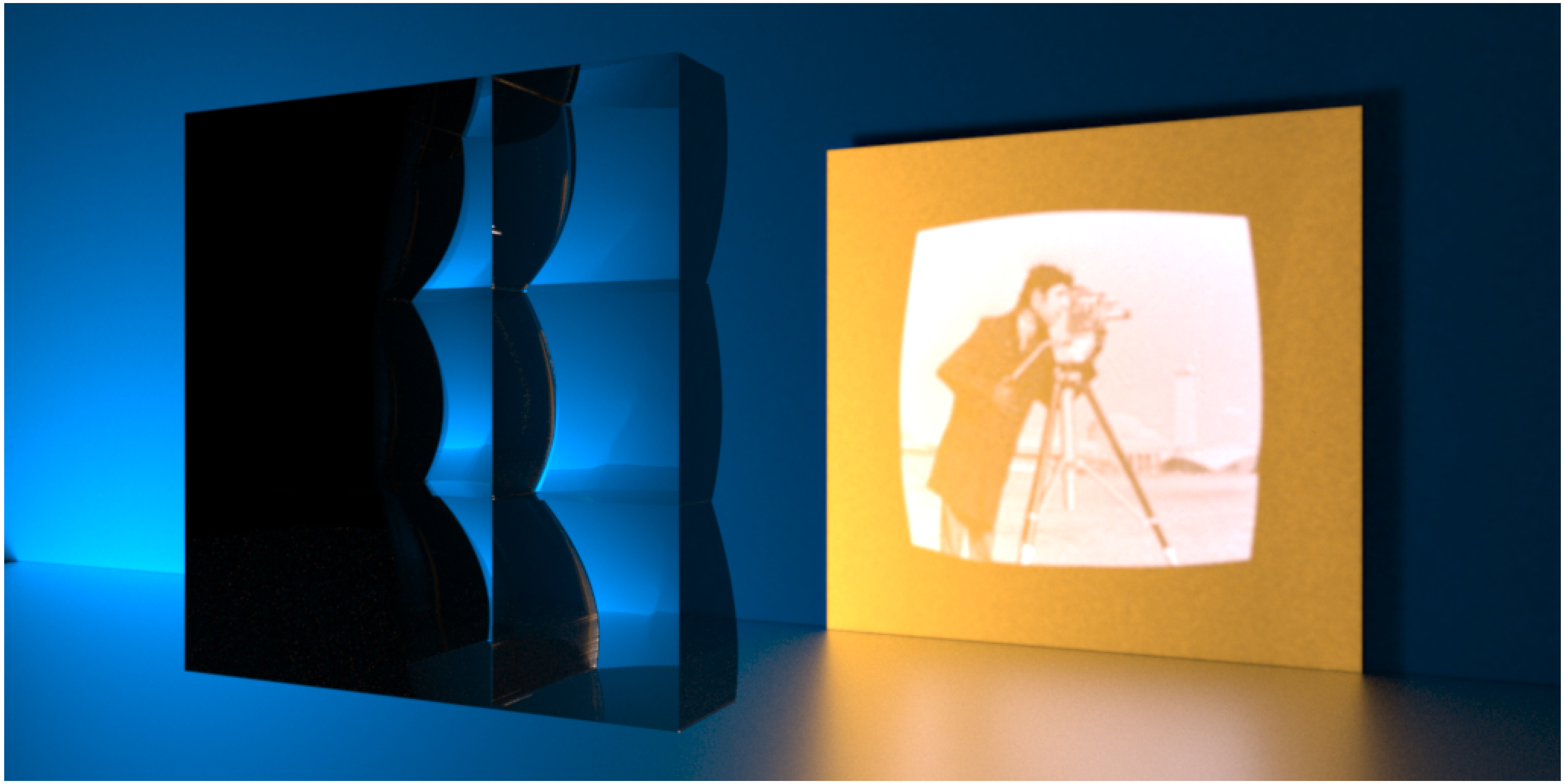


$\mathcal{R}_T$

$\mathcal{R}_T$ for a uniform source

Pillows

Goal: decompose the optical component into several smaller *pillows*



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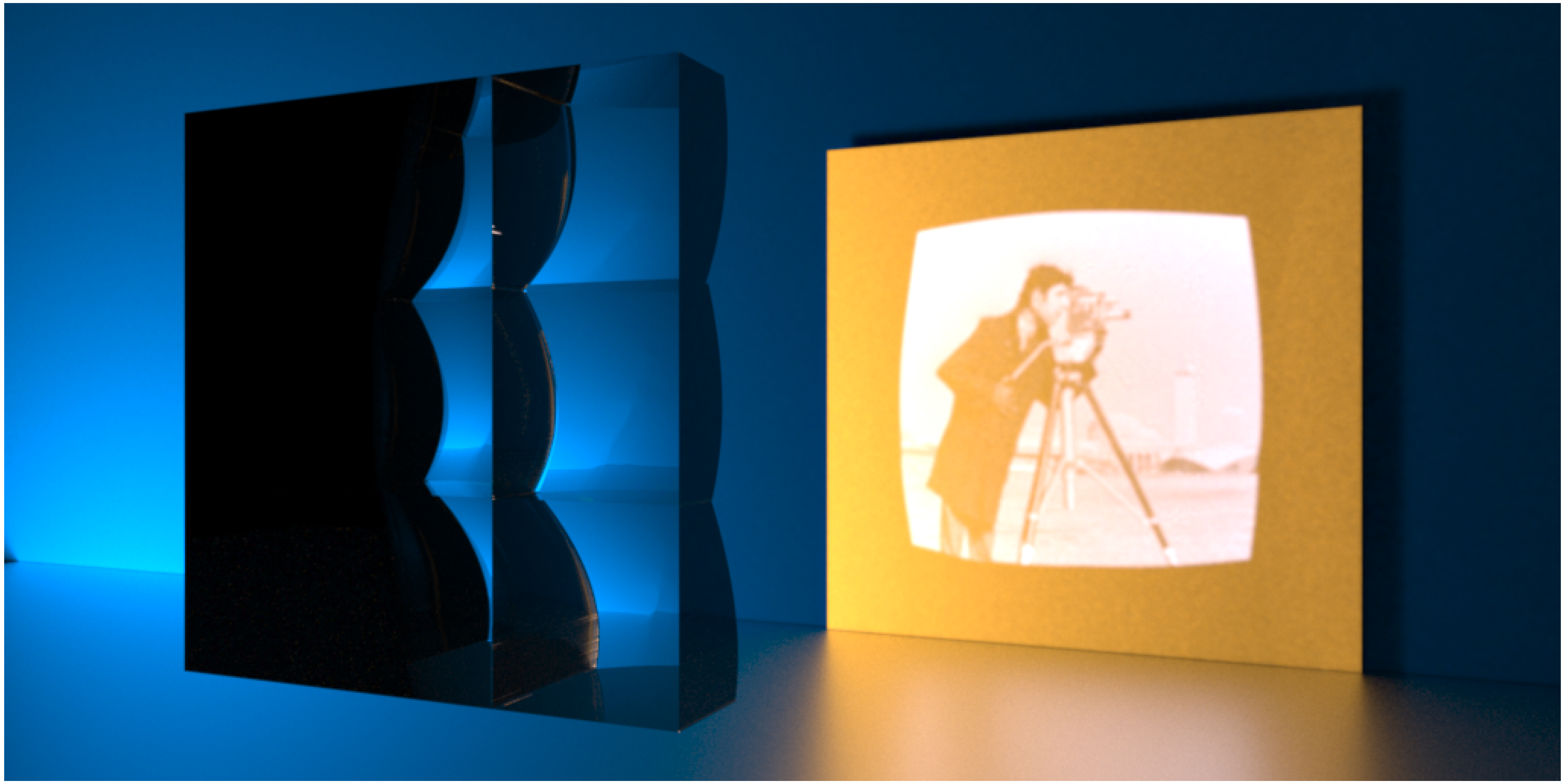
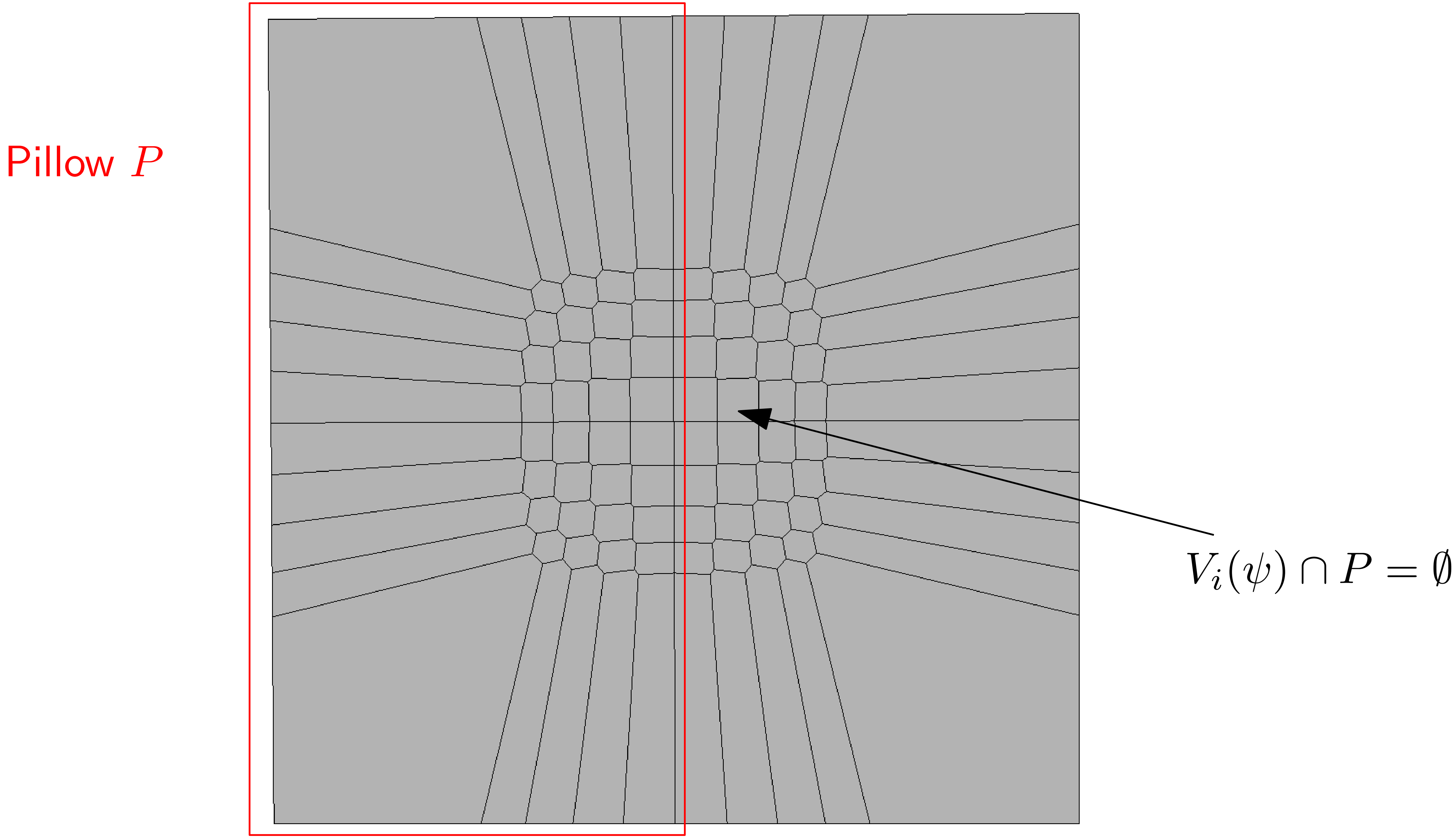


Problem: support of the pillow is *small* $\Longrightarrow$ choice of the initial weights

Solution: interpolation between two source densities

Pillows

Interpolation: pillow = left part of the plane



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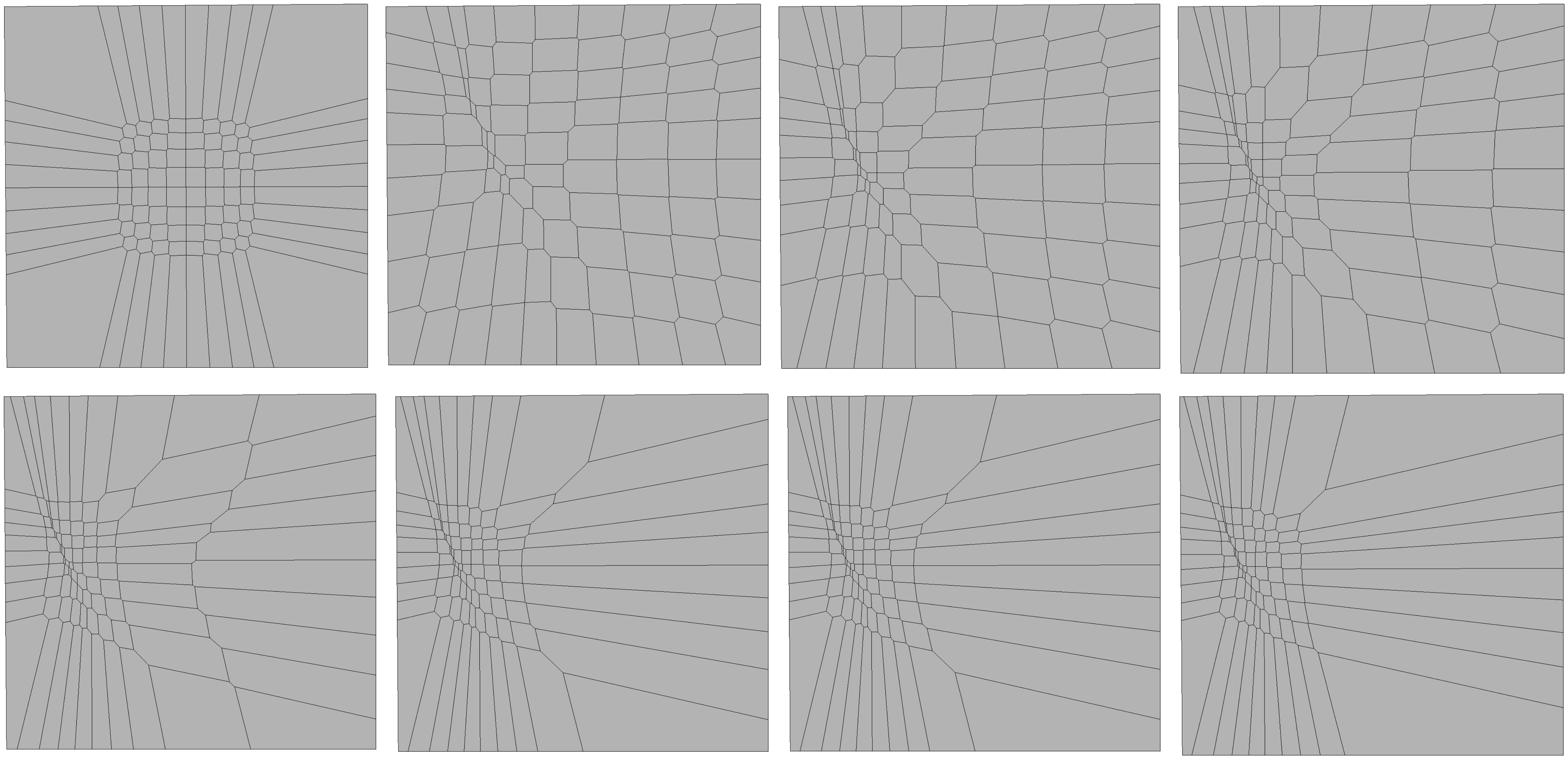

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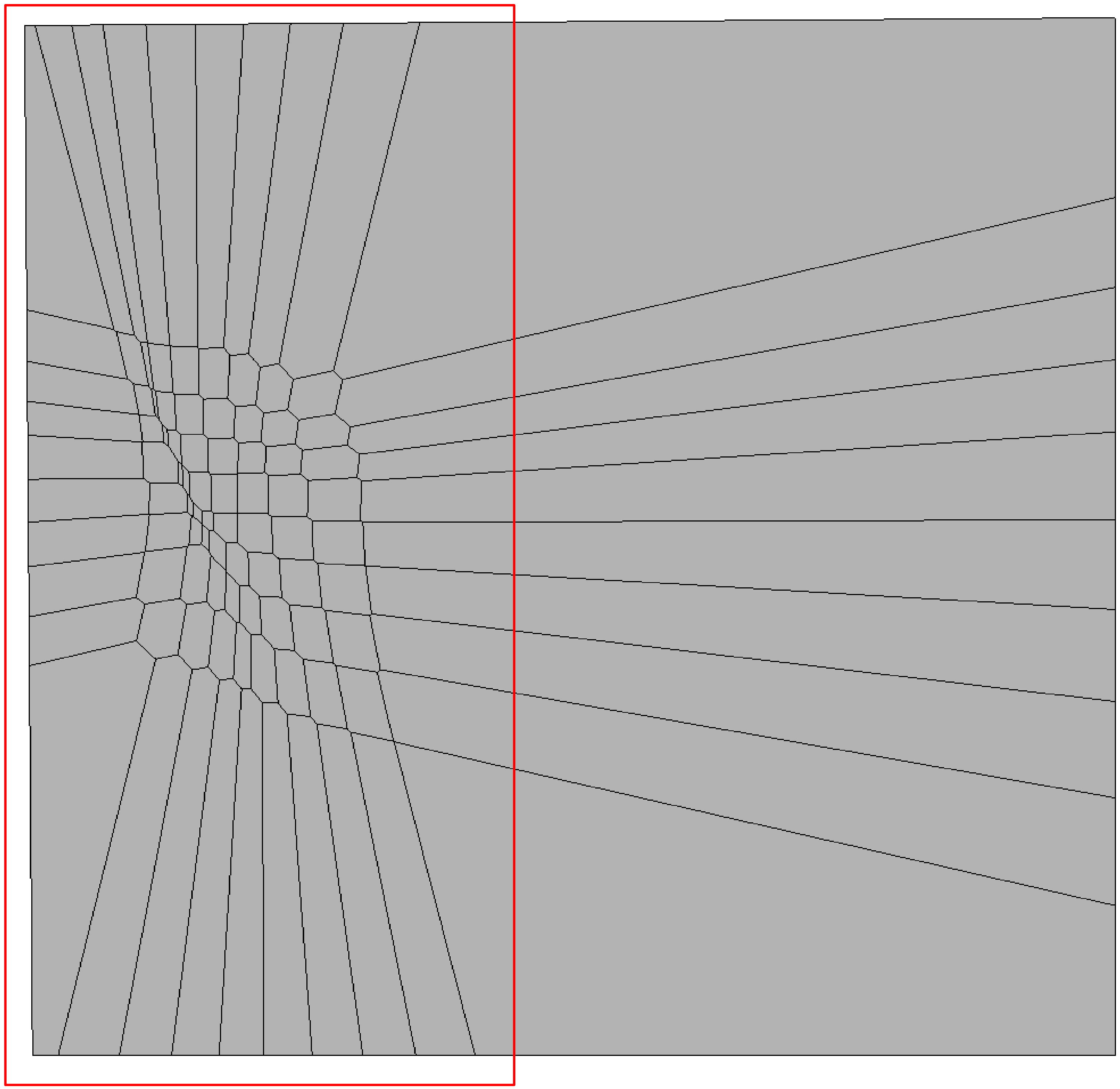
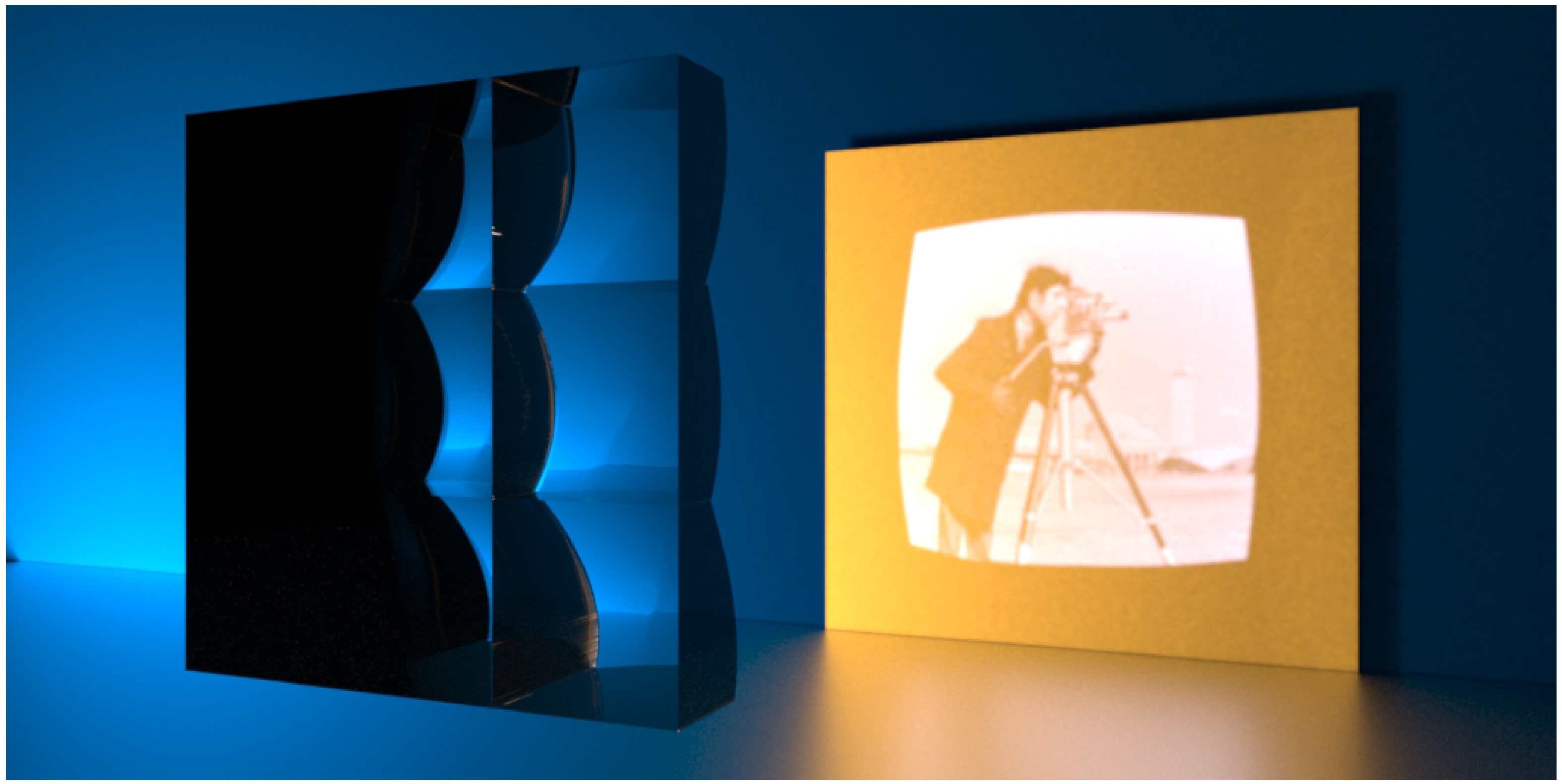
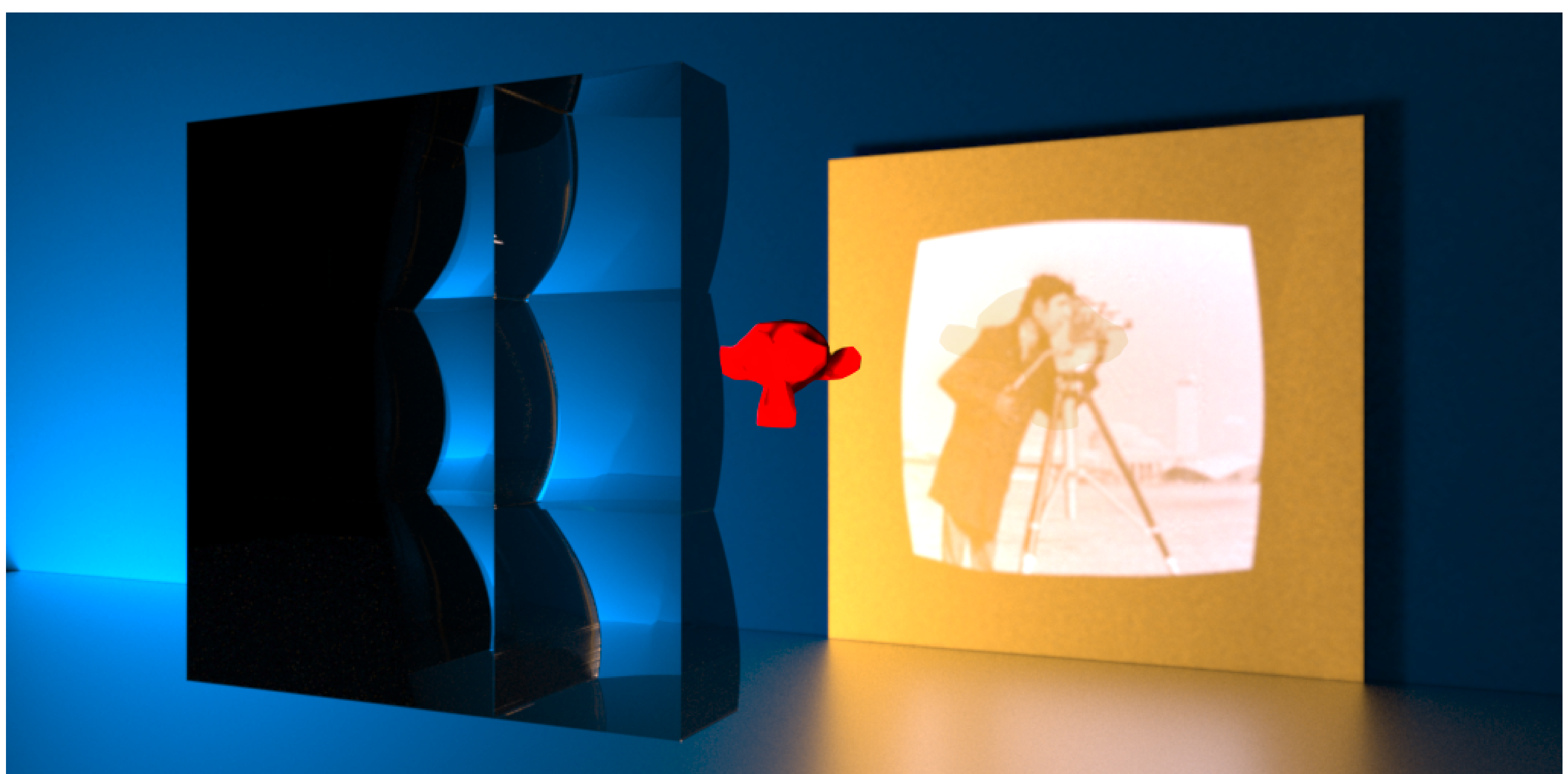


Pillows

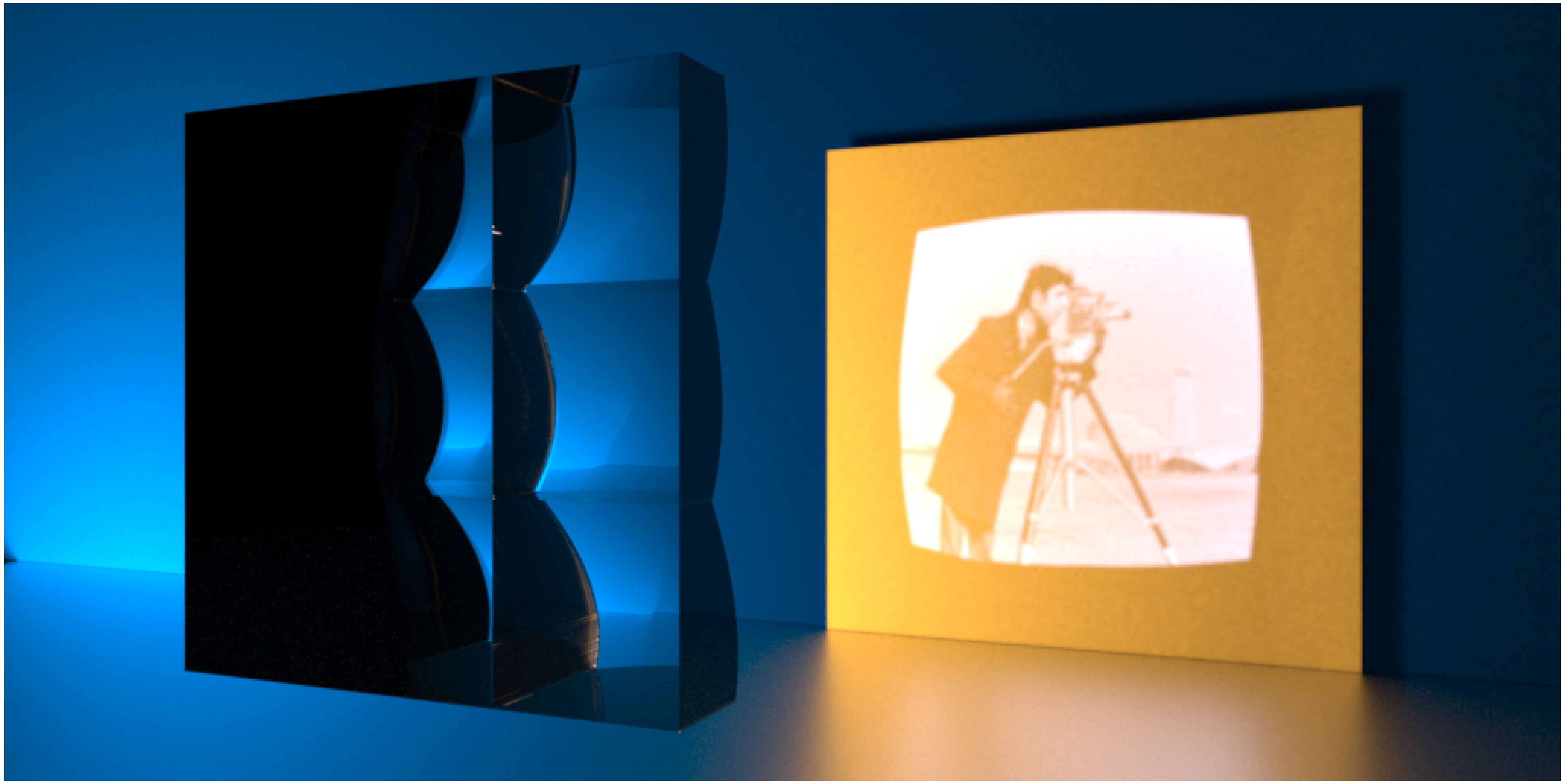
Example: lens made of 9 pillows, without and with an obstacle





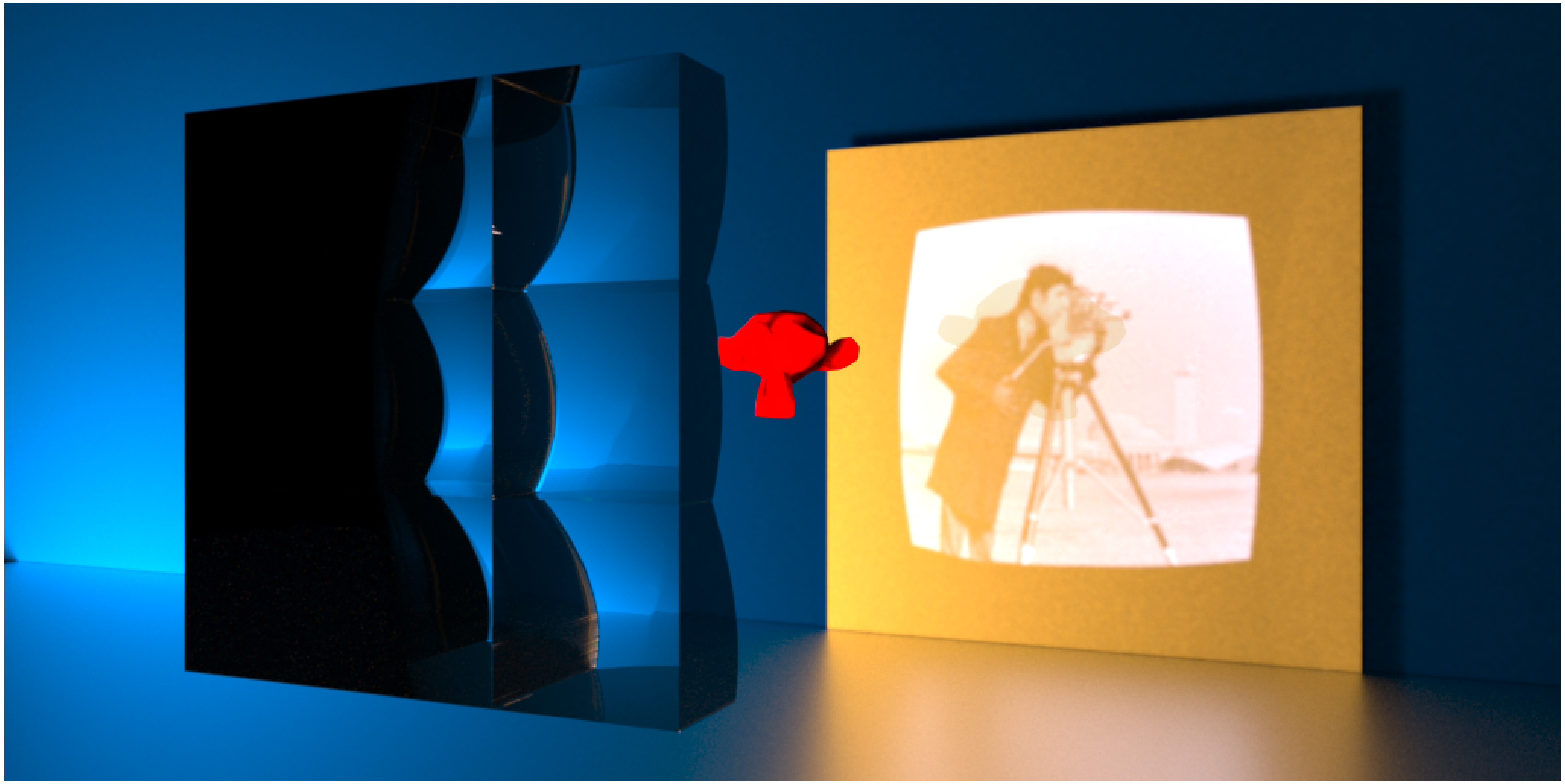
Pillows

Without an obstacle

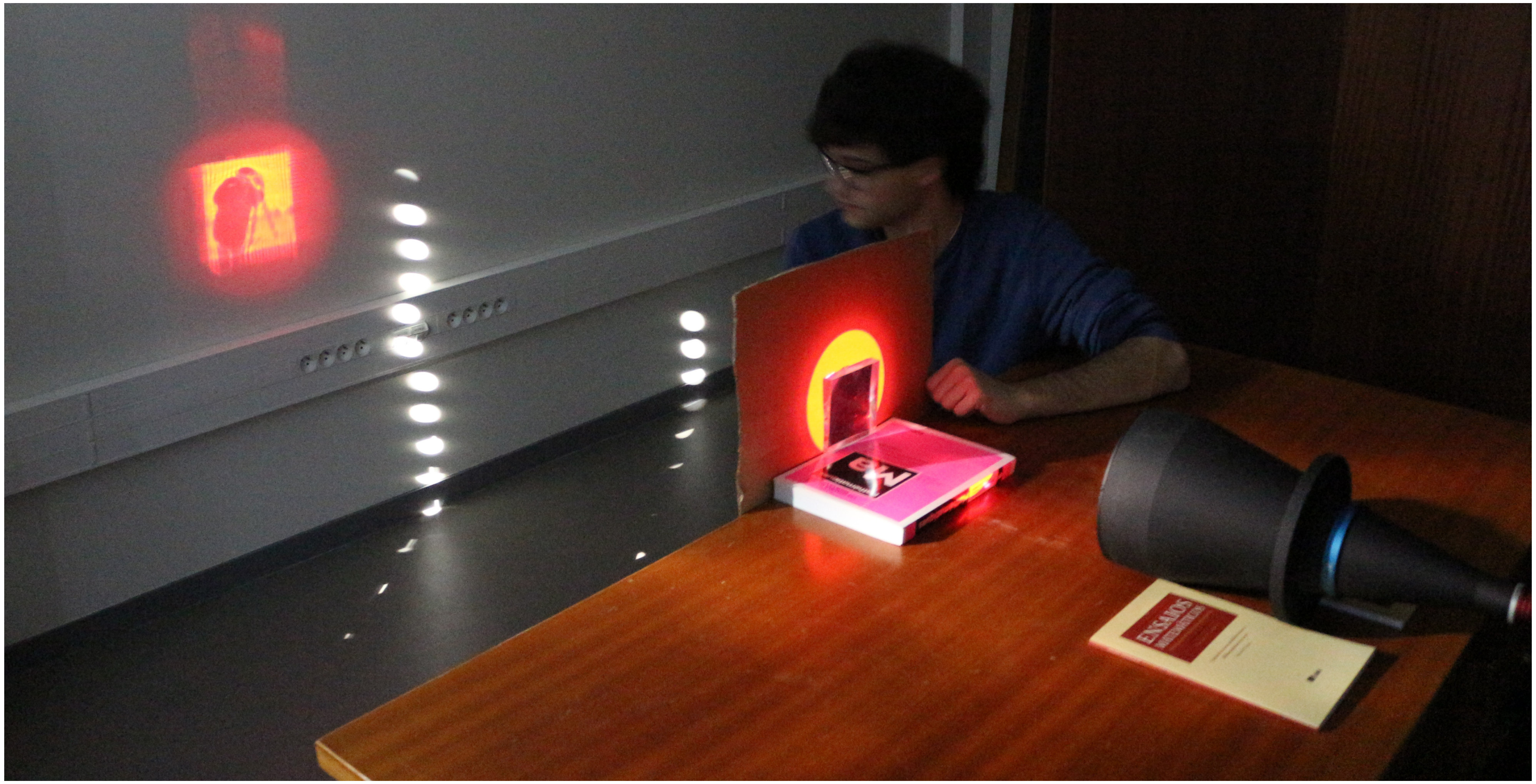


Pillows

With an obstacle



Demo



Conclusion & Perspectives

We saw

- a *general framework* to solve 8 different optical component design probems,
- and a *generic* and *efficient* algorithm able to solve them.

Code: OT between a density supported on a triangulated surface and
a discrete measure on a 3D point cloud for the quadratic cost

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- Near-Field: optimal transport replaced by a *prescribed Jacobian*
- extended sources
- initialization strategies

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Thank you for you attention.

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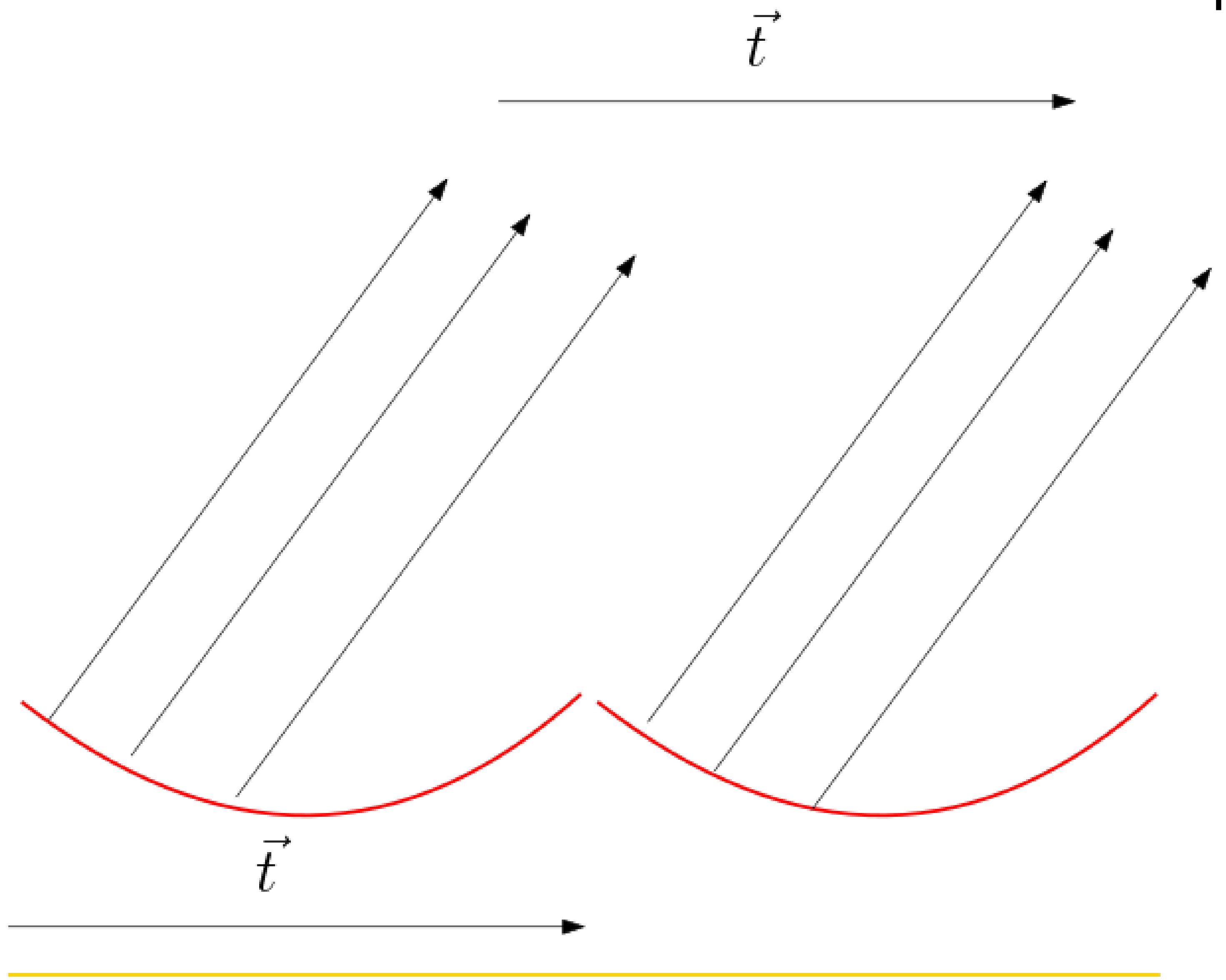
Pillows (bis)

Image alignment: we can *not* use the Far-Field assumption



$\Longrightarrow$ we use an iterative method to simulate a *Near-Field* target