Programme des JGA2017
(Centre Paul Langevin, Aussois, 11-15 décembre 2017)

	December 11	December 12	December 13	December 14	December 15
8h30-9h00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9h00-9h30	Opening session	Jean Cardinal	Günter Rote	Günter Rote	Jérémy Cochoy
9h30-10h00	Irène Waldspurger				Nicolas Berkouk
10h0-10h30		Vincent Pilaud			Mathieu Carrière
10h30-11h00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11h00-11h30	Jean Cardinal	Irène Waldspurger	Louis Theran	Louis Theran	Iordan Iordanov
11h30-12h00					Jocelyn Meyron
12h00-12h30	Xavier Goaoc	Theo Lacombe	Mathias Lepoutre	Lucas Isenmann	Thomas Magnard
	Lunch	Lunch	Lunch	Lunch	Lunch
	break	break	break	break	
15h30-16h00		Social event			
16h00-16h30	Irène Waldspurger	(excursions)	Nicolas Bonichon	Nicolas Bonichon	
16h30-17h00					
17h00-17h30	Coffee break		Coffee break	Coffee break	
17h30-18h00	Jean Cardinal	Coffee break	Louis Theran	André Lieutier	
18h00-18h30				Mathijs Wintraecken	
18h30-19h00	Aperitif		Business Meeting	Olivier Devillers	
19h00-19h30				Ruqi Huang	
19h30-20h00	Dinner	Special	Dinner	Dinner	
20h00-20h30		dinner			

Mini-cours (orateurs invités)

Nicolas Bonichon

Quelques résultats sur les spanners géométriques

Un graphe géométrique est un graphe défini à partir d'un nuage de points (par exemple dans le plan). L'étirement d'un graphe géométrique est le pire rapport entre la distance dans le graphe et la distance Euclidienne, pour toute paire de points du graphe. Un t-spanner est un graphe (ou une famille de graphes) dont l'tirement est born par t. Dans cet exposé je présenterai quelques résultats relatifs à certains spanners: triangulations de Delaunay, Θ-graphs, spanners de degré borné, routage dans les triangulations de Delaunay...

Jean Cardinal

The geometry of 3SUM, k-SUM, and related problems

We consider the 3SUM and k-SUM problems, defined as fixed-parameter versions of the Subset Sum Problem: Given a set of n numbers, does there exist a subset of k of them whose sum is 0? The 3SUM problem was first considered by computational geometers as a way to argue of quadratic lower bounds for some natural geometric problems. It has since become a cornerstone of the so-called fine-grained complexity theory, dealing with the computational complexity of problems in P. We will review the notion of 3SUM-hardness and the relation between k-SUM and other problems. Then we will consider recent attempts to beat the quadratic barrier for 3SUM-hard problems. Finally, we will consider the design of efficient linear decision trees for k-SUM. All these topics heavily rely on geometric notions, such as dominance detection and hyperplane arrangements.

Gunter Rote
The Computational Geometry of Congruence Testing

Part I. Testing two geometric objects for congruence, i.e., whether they are the same up to translations and rotations (and possibly reflections) is a fundamental question of geometry.

In the first part, I will survey the various algorithmic techniques that have been used since the 1970s to solve the problem in two and three dimensions in $O(n \log n)$ time for two n-point sets, such as string matching, planar graph isomorphism (Sugihara [6]), and the reduction technique of Atkinson [3].

In d-dimensions, for small constant d, the best previous algorithm takes $O(n^{\lceil d/3 \rceil} \log n)$ time (Brass and Knauer [4]). There is also a randomized Monte Carlo algorithm of Akutsu [1] and Matoušek, which takes $O(n^{\lfloor d/2 \rfloor/2} \log n)$ time but which may miss to find a congruence, with small probability. I will review the involved techniques: the basic dimension reduction technique of Alt, Mehlhorn, Wagener, and Welzl [2], the canonical forms of Akutsu [1], the closest-pair graph of Matoušek.

Part II. In the second part, I will introduce our recent algorithm for solving the 4-dimensional problem in $O(n \log n)$ time (joint work with Heuna Kim [5]). This algorithm will require the study of four-dimensional geometry, in particular the structure of four-dimensional rotations, Hopf fibrations, and the regular polytopes.

References

[1] T. Akutsu. On determining the congruence of point sets in d dimensions. *Computational Geometry: Theory and Applications*, 4(9):247–256, 1998.

[2] H. Alt, K. Mehlhorn, H. Wagener, and E. Welzl. Congruence, similarity, and symmetries of geometric objects. *Discrete & Computational Geometry*, 3(1):237–256, 1988.

[3] M. D. Atkinson. An optimal algorithm for geometrical congruence. *Journal of Algorithms*, 8(2):159–172, 1987.

[4] P. Brass and C. Knauer. Testing the congruence of d-dimensional point sets. *International Journal of Computational Geometry and Applications*, 12(1&2):115–124, 2002.

[5] H. Kim and G. Rote. Congruence testing of point sets in 4-space. In S. Fekete and A. Lubiw, editors, *32st International Symposium on Computational Geometry (SoCG 2016)*, volume 51 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 48:1–48:16, 2016. Full version in arXiv:1603.07269 [cs.CG].

[6] K. Sugihara. An $n \log n$ algorithm for determining the congruity of polyhedra. *Journal of Computer and System Sciences*, 29(1):36–47, 1984.

Louis Theran

Global rigidity and graph realization

The global rigidity problem asks whether, given a configuration of n points in d-dimensional Euclidean space and a set of m pairs of points, there is another non-congruent configuration where the distances between the m specified pairs are the same.

The graph realization problem asks whether, given a graph G with n vertices and a desired edge length for each of its m edges, there is a configuration of n points in d-dimensional Euclidean space so that the distances between the endpoints of each edge are the desired ones.

Each of these problems is NP-hard, but under certain genericity assumptions they become tractable. This mini-course will introduce the topic and cover recent progress on both problems.

Irène Waldspurger

Algorithms for phase retrieval problems

A phase retrieval problem consists in recovering an unknown element in some complex vector space from the modulus of linear measurements. Such problems naturally arise in particular in molecular imaging and audio processing. The goal of this mini-course is to give an overview of some algorithms that have been designed to solve them, with a focus on the ones for which rigorous correctness guarantees exist.

First part: : we will introduce phase retrieval problems and their applications. We will describe conditions under which these problems can be expected to be well-posed.

Second part: the non-convexity of phase retrieval problems makes them hard to solve. This issue can be partially overcome with convexification techniques. We will present two algorithms that use these techniques in different ways: PhaseLift and PhaseMax.

Third part: more recently, it has been noticed that, in some settings, phase retrieval problems can also be solved with simpler, non-convex, heuristics. We will describe a few of these heuristics, and explain how to derive correctness guarantees for them.

Exposés courts (participants)

Nicolas Berkouk

Stable resolutions of multi-parameter persistent modules

The problem of classifying persistent module with many parameters is known to be very hard. Multi-graded betti numbers, one of the only invariant we have to study such modules, are highly not stable with respect to the interleaving distance. However, they arise from the existence of free minimal resolutions and we propose to show how we can equip the derived category of persistent modules (the one in which resolutions *naturally* live) with a derived distance to make those resolutions stable.

In the future, this might be of help to build robust, easy calculable invariants to understand multi-parameter persistent modules.

Mathieu Carrière

A theoretical framework for the analysis of Mapper

Mapper is probably the most widely used TDA (Topological Data Analysis) tool in the applied sciences and industry. Its main application is in exploratory analysis, where it provides novel data representations that allow for a higher-level understanding of the geometric structures underlying the data. The output of Mapper takes the form of a graph, whose vertices represent homogeneous subpopulations of the data, and whose edges represent certain types of proximity relations. Nevertheless, the inherent instability of the output and the difficult parameter tuning make the method rather difficult to use in practice. This talk will focus on the study of the structural properties of the graphs produced by Mapper, together with their partial stability properties, with a view towards the design of new tools to help users set up the parameters and interpret the outputs.

Jérémy Cochoy

Endow persistent multi-modules with a structure of algebra

Persistence is a tool which allow to extract topological informations from point clouds, graphs, and other type of dataset at multiple scales. Persistent (multi)modules are algebraic modules over a ring that arise naturally when computing the persistent (and multipersistent) homology. It contain the homology of each step of the filtration of the data, and the relation between them through morphisms. Using the cup product, we will see how we can add to this module a multiplicative structure, which allow them to retain even more informations on the data.

Olivier Devillers

Walking in Poisson Delaunay triangulations

The talk will review some results about several walking strategies between vertices of the Delaunay triangulation of points distributed according Poisson law of large intensity n.

We provide a first non-trivial lower bound for the distance between the expected length of the shortest path between (0,0) and (1,0). Simulations indicate that the correct value is about 1.04. We also prove that the expected length of the so-called upper path converges to $35/(3\pi^2)$, giving an upper bound for the expected length of the smallest path.

In dimension d, we prove that the expected length of the Voronoi path between two points at distance 1 in the Delaunay triangulation associated with X is $\sqrt{(2d/\pi)} + O(1/\sqrt{d})$ when d goes to infinity. In any dimension, we provide a precise interval containing the actual value; in 3D the expected length is between 1.4977 and 1.50007.

Joint work with Nicolas Chenavier, Louis Noizet, and Pedro M. M. de Castro.

Xavier Goaoc

Fonctions de pulvérisation d'hypergraphes (géométriques)

In combinatorial and computational geometry, the complexity of system of sets is often studied via its shatter function. I will discuss how the asymptotic growth rate of a shatter function is governed by a single of its values, in the spirit of the classical *Vapnik-Chernonenkis dimension* of hypergraphs. In particular, I will describe a probabilistic construction that refutes a conjecture of Bondy and Hajnal. This is joint work with Boris Bukh (arxiv preprint).

Ruqi Huang

Adjoint Map Representation for Shape Analysis and Matching

Operator-based representations, such as functional maps and shape difference operators, have been more and more widely used in geometry processing. In this talk, I will introduce the adjoint representation of functional maps, which can be seen as a complementary of the aforementioned operators. Especially, I will demonstrate the compactness and informativeness of the adjoint representation, and how it helps in applications such as bi-directional shape matching, shape exploration, and pointwise map recovery.

Iordan Iordanov

Implementing Delaunay triangulations of the Bolza surface

The CGAL library offers software packages to compute Delaunay triangulations of the (flat) torus of genus one in two and three dimensions. To the best of our knowledge, there is no available software for the most symmetric extension, i.e., the Bolza surface, a hyperbolic manifold homeomorphic to a torus of genus two.

In this presentation, I will give an overview of our implementation, based on the theoretical results and the incremental algorithm proposed last year by Bogdanov, Teillaud, and Vegter at SoCG 2016. This is a joint work with Monique Teillaud, and was presented at SoCG 2017 in Brisbane, Australia. I will describe the representation of the triangulation, as well as details on the different steps of the algorithm and the algebraic complexity of the predicates, and I will present experimental results.

Lucas Isenmann

Dushnik-Miller dimension of TD-Delaunay complexes

TD-Delaunay graphs, where TD stands for triangle-distance, are obtained from a variation of Delaunay triangulation consisting in replacing circles by homothetic triangles. It was noticed that every triangulation is the TD-Delaunay graph of a set of points in R^2, and conversely. It seems natural to study the generalization of this property in higher dimensions. Such a generalization is obtained by replacing equilateral triangles by regular simplexes in dimension R^d. The abstract simplicial complexes obtained from a TD-Delaunay complex in dimension d are of Dushnik-Miller dimension d + 1. The converse holds for d = 2 and 3 and it was conjectured to hold for larger d. Our work with Daniel Gonalves is to disprove the conjecture already for d = 4.

Theo Lacombe

Smoothed optimal transport: fast computation of matching distances and other applications

Computing matching distances between point clouds is known to be a difficult, often intractable, problem. However, these problems can be formulated in the framework of optimal transport theory which has known significant improvement in recent years from a numerical perspective. The entropic smoothing is a relaxation method which provides a numerically efficient estimation of optimal transport/matching cost which has very interesting practical properties from a statistical and learning perspective. We will present some applications of smoothed optimal transport to matching cost computation and statistics on persistence diagrams.

Mathias Lepoutre

A bijective proof of the enumeration of maps in higher genus

Bender and Canfield proved in 1991 that the generating series of maps in higher genus is a rational function of the generating series of planar maps. In this talk, I will give the first bijective proof of this result. Our approach starts with the introduction of a canonical orientation that enables us to construct a bijection between 4-valent bicolorable maps and a family of unicellular blossoming maps.

André Lieutier

The reach, metric distortion, geodesic convexity and the variation of tangent spaces

The reach of a closed set in euclidean space is the infimum of distances between the set and its medial axis. In this talk we discuss three results. The first two concern general sets of positive reach: We first characterize the reach by means of a bound on the metric distortion between the distance in the ambient Euclidean space and the geodesic distance in the set of positive reach. Secondly, we prove that the intersection of a ball with radius less than the reach with the set is geodesically convex, meaning that the shortest path between any two points in the intersection lies itself in the intersection. For our third result we focus on manifolds with positive reach and give a bound on the angle between tangent spaces at two different points in terms of the distance between the points and the reach. Joint work with Jean-Daniel Boissonnat and Mathijs Wintraecken.

Thomas Magnard

Topological embedding of graphs into 2-complexes

We consider the problem of deciding whether an input graph G admits a topological embedding into a two-dimensional simplicial complex C. This problem includes, among others, the embeddability problem of a graph on a surface and the topological crossing number of a graph, but is more general.

We give a polynomial-time algorithm if the complex C is fixed. The strategy is to reduce the embeddability problem to the following decision problem: Given a surface S, a graph G and a subgraph of G embedded on S, decide whether this embedding can be extended to an embedding of the greater graph.

Jocelyn Meyron

Geometric methods for the conception of components in non-imaging optics

We will present a generic and parameter-free algorithm to efficiently build a wide variety of optical components, such as mirrors or lenses, that satisfy some light energy constraints. In all of our problems, one is given a collimated or point light source and a desired illumination after reflection or refraction and the goal is to design the geometry of a mirror or lens which transports exactly the light emitted by the source onto the target.

We propose a general framework and show that many optical component design problems amount to solving a *Light Energy Conservation* equation that involves the computation of *Visibility diagrams*. We show that these diagrams all have the same structure and can be obtained by intersecting a 3D Power Diagram with a planar or spherical domain. This allows us to propose an efficient and fully generic algorithm. Our solutions can satisfy design constraints such as convexity or concavity and are always graphs above the plane or the sphere. We show the effectiveness of our algorithm on numerous numerical examples.

Vincent Pilaud
Quotientopes

The weak order is a fundamental lattice on permutations. It is obtained by orienting the permutahedron in a linear direction. The quotients of this lattice are often interesting: for instance, the Tamari lattice on binary trees is the quotient of the weak order by the sylvester congruence. Nathan Reading studied in detail the combinatorics and geometry of lattice quotients of the weak order (and more generally of the poset of regions of an hyperplane arrangement). He showed in particular that for any lattice congruence of the weak order, the cones obtained by glueing together the regions of the Coxeter arrangement that belong to a same equivalence class form a complete fan. In this talk, we will show that this fan is the normal fan of a polytope called quotientope. Joint work with Francisco Santos.

Mathijs Wintraecken
Reach comparison theory

To be able to triangulate Riemannian manifolds with submanifolds or more generally stratified manifolds we need to understand the geometry of submanifolds. Let S be a submanifold of M. For simplicity we assume that S lies completely in a convex geodesic ball of M. The reach of S as a subset of M can be defined in much the same way as in the Euclidean setting. We discuss how the reach of the inverse image of S under the exponential map at a point p near S compares to the reach of S as a subset of M.